UNIT 12 PROBABILITY

Structure

12.1 INTRODUCTION

In the previous unit you studied some basic concepts in statistics, which is concerned with real life observation or experimentation. In this unit we will talk about probability theory which deals with the same type of experimentation and observation at a hypothetical level. The two disciplines are, therefore, closely connected. In fact, the methodology and techniques of statistics, particularly those dealing with sample data, derive their theoretical basis and justification from probability theory. In this unit, we discuss the elementary principles and concepts of probability, which will lead to a better understanding of statistical methods.

Probability theory had its beginnings in the gambling houses of France in the late twelfth century. Some of the problems of gambling were brought to the attention of mathematicians like Pascal and Fermat by an aristocratic gambler. They and other mathematicians in Europe were quick to notice that various problems of gambling and betting could be tackled through methods of permutation and combination, which you have studied in Unit 3. They also realised that some other problems of chance variation needed other techniques of mathematics like calculus. They also foresaw the enormous possibilities of practical applications of the laws of chance which they had derived, in various fields of science and human activity. For example, these laws could be applied to the study of errors of observation and measurement, the study of variations in unpredictable phenomena like the number of deaths from a disease, and so on. It is, however, only in the 20th century that the basic concepts and principles of probability theory have been given a completely mathematical basis by mathematicians like Kolmogorov and Markov, among others.

In this unit, we discuss the meaning of probability and why it is useful in statistics. We also discuss how probabilities of events can be calculated using certain basic theorems and results. These concepts and techniques will be used while discussing the frequency distributions in Units 13 and 14.

Objectives

After reading this unit, you will be able to

- define random experiment, event and sample space in the context of probability
- explain the meaning of the probability of an event
- state certain fundamental theorems of probability
- calculate probabilities of events using these theorems.

12.2 PRELIMINARIES

Let's first get familiar with some terms which we shall be using in this unit. Suppose you toss a coin. You will get a head or a tail. Suppose you get a head when you toss it for the first time. In the language of probability you have performed an experiment. The process of tossing the coin and making the observation that you got the head or the tail is called an **experiment.** The result of the experiment or the observation that you made after the experiment is performed is called an **outcome**. The outcome of your experiment above was 'head'.

Let's take another example. Suppose you pick one card from a deck of cards and find that it is the six of spades. Picking a card and noting what you get is the experiment, and six of spades is the outcome.

We have talked of two experiments here. You will agree that in tossing a coin, the total number of possible outcomes is two; the head (H) and the tail (T). Similarly, the total number of possible outcomes in the experiment of picking a card from a deck of cards is fifty-two (see Fig. 1).

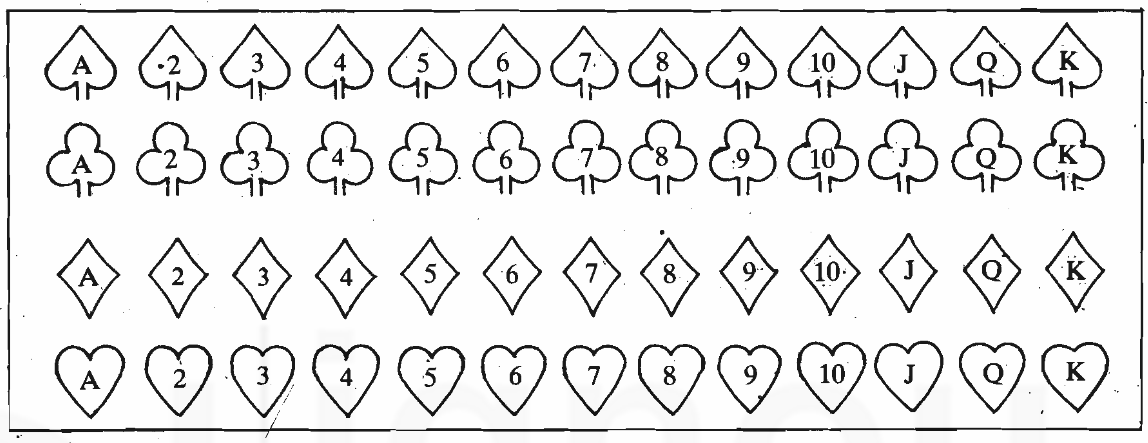


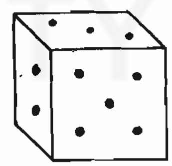
Fig. 1: Sample space when a card is drawn from a normal pack of cards

The set of all possible outcomes of an experiment is called the **sample space,** and is denoted by S or Ω.

Thus, in the experiment of tossing a coin, the sample space is (H,T), and in the experiment of picking a card, the sample space is the set of all cards in the deck.

Try this exercise now.

E1) You roll a die and note that a 3 comes up.



a) What is the experiment?
b) What is the outcome?
c) What is the total number of outcomes?
d) What is the sample space?

In all the examples which we have seen till now, the sample space consisted of isolated points, which we could list. But suppose we consider an experiment which consists of measuring the "lifetimes" of electric bulbs produced by a company. The outcome of this experiment is a time t (in hours), which lies in some interval, say $0 \leq t \leq 4000$. Here we have assumed that no bulb lasts longer than 4000 hours. The points of the sample space of this experiment cannot be written as isolated points. In fact, sample sapce $S = [0,4000]$. Such sample spaces which consist of one or more intervals are called **continuous sample spaces.** On the other hand, those sample spaces which consist of isolated points are called **discrete sample spaces.** You can compare these definitions of continuous and discrete sample spaces with the definitions of continuous and discrete random variables given in Unit 11, and note the points of similarity.

Sometimes we may not be interested in all the points of a sample space, or in all possible outcomes. For example, in rolling a die, we may be interested only in odd numbers (say). This means the only outcomes of interest to us are 1, 3 and 5. We call each one of the outcomes 1, 3 or 5, a **favourable outcome.**

The elements of the sample space or the individual outcomes associated with any experiment are called the **elementary events** of that experiment. Thus, 1, 2, 3, 4, 5 and 6 are the elementary events of the experiment of rolling a die. In general, any subset of the sample space is called an **event**. So, {1, 3, 5} denotes the event of getting an odd number after rolling a die. The outcomes 1, 3 and 5 are favourable to this event.

Now, we are ready to give the definition of probability. We shall define it separately for finite and infinite sample spaces.

12.2.1 Probability in a Finite Sample Space

In this sub-section we shall define the probability of an event in a finite sample space.

Definition 1: The probability of an event A, P(A), is given by

$$P(A) = \frac{\text{number of outcomes favourable to A}}{\text{number of all possible outcomes}}$$

If we apply this definition to the event A: "getting an odd number when a die is rolled", then we get $P(A) = 3/6 = 1/2$.

This definition of probability is also called the **classical definition.** Apart from this there is another way in which we can define probability. This approach is called the **empirical** or the **relative frequency** approach. Let's see what it is.

Suppose we throw a die 600 times, and suppose 98 times we get a '2'. Then the relative frequency of the occurrence of 2 is 98/600, and we can take this to be a reasonable estimate of its probability. Thus, we can also define the probability of an event A as:

$$P(A) = \frac{\text{number of observed favourable outcomes}}{\text{total number of observed outcomes}}$$

You will notice that the only difference between the classical and the empirical definition of P(A) lies in the word "observed". This word indicates that we have actually performed the experiment a number of times before arriving at a value of P(A). We shall now illustrate the concept of probability through an example.

Example 1: Find the probabilities of each of the following events, when one card is picked from a well-shuffled deck of cards.

A : getting a queen
B : getting the jack of hearts
C : getting a red card

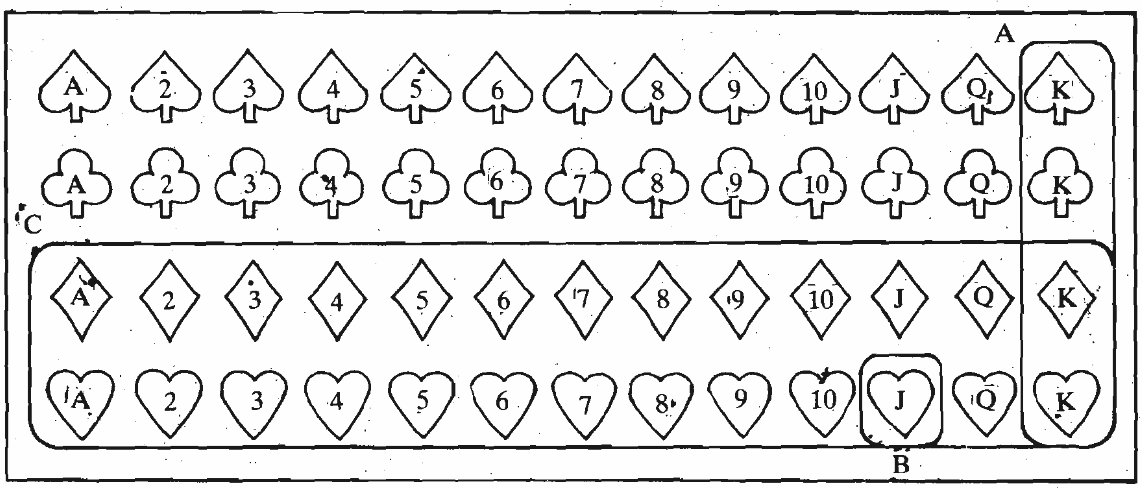


Fig. 2

From Fig. 2 you will find that there are

i) 4 outcomes favourable to A,
ii) 1 outcome favourable to B, and
iii) 26 outcomes favourable to C, out of a total of 52 possible outcomes.

Thus, P(A) = 4/52 = 1/13,
P(B) = 1/52, and
P(C) = 26/52 = 1/2.

This example also suggests the use of Venn diagrams to represent the sample space and the events. You have already studied Venn diagrams in Unit 1. We consider the sample space of an experiment as the universal set, and represent it by a rectangle. Various events related to this experiment are subsets of the sample space, and are represented by closed regions within this rectangle. We follow the same rules as for the Venn diagrams of sets.

Thus,

i) $A \cap B$ consists of those outcomes which are favourable to both A and B.
ii) $A \cup B$ consists of outcomes which are favourable to A, or B, or both.
iii) A′ or $\bar{A}$ consists of all those outcomes which are not favourable to A. Also see Fig. 3.

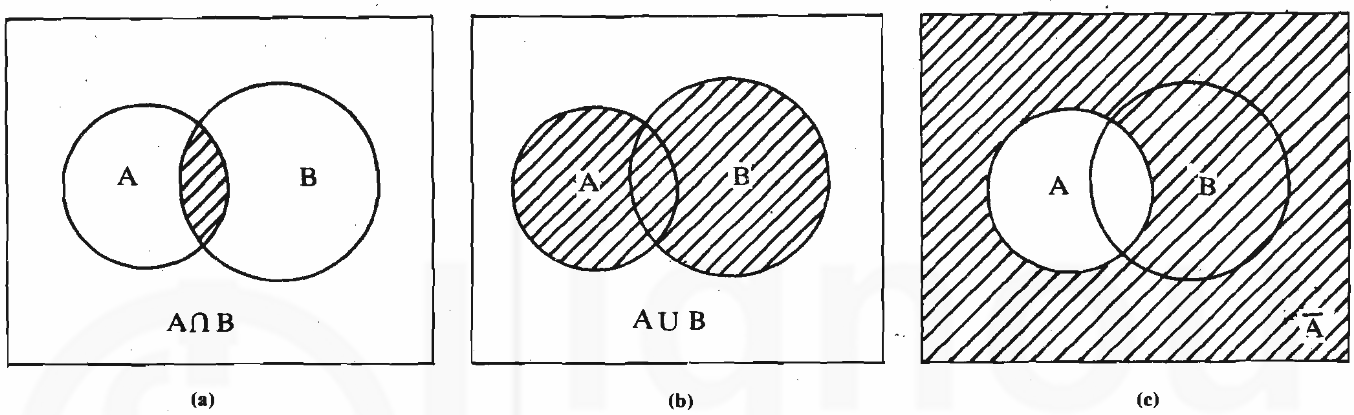


Fig. 3: The shaded region shows (a) $A \cap B$ (b) $A \cup B$ (c) A

Now, P(A) = 0 means that there are no outcomes favourable to A. We can also say that A is an impossible event.

On the other hand, P(A) = 1 would mean that the number of outcomes favourable to A is the same as the total number of outcomes. That is, A is the entire sample space. This means that the event A is sure to happen. Now, an event is a subset of the sample space.

Thus, given any event A, we can write $\phi \subseteq A \subseteq S$.

This means that for any event A, we have $0 \leq P(A) \leq 1$.

Try to do these exercises now.

E2) Two dice are thrown simultaneously. What is the probability that the sum of the numbers on the dice is 7?

E3) Several feminist groups accused an organisation of discriminating against its female employees. According to these groups, women were not being given sufficient opportunities for promotions to high level executive positions. The following table gives the available data on the organisation's promotions.

Year	Number of women promoted	Number of men promoted
1984	5	15
1985	6	16
1986	10	8
1987	8	10
1988	10	10

If an employee who was promoted during these years is selected at random, what is the probability that the employee is a woman?

We have seen how to calculate the probabilities of events belonging to a finite sample space. But this method cannot be extended to the events of an infinite sample space. In the next sub-section we'll see how to obtain the probabilities of events in an infinite sample space.

12.2.2 Probability in an Infinite Sample Space

It is impossible to apply our definition of probability to the probabilities P_w of individual sample points w, where the sample space S is not finite. The only alternative in this case is to start with a set of events $\{A_1, A_2, \ldots\}$ of S such that their probabilities are known, and then to obtain probabilities of more complex events (which are combinations or compounds of $A_1, A_2, \ldots$) on the assumption that the probabilities satisfy

i) $0 \leq P(A) \leq 1$

ii) $P(S) = 1$

iii) $P(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$ if $A_1, A_2, \ldots$ are disjoint events, that is, if

$P(A_i \cap A_j) = 0$ for $i \neq j$.

These relations are regarded as postulates or axioms, which are satisfied by the probabilities of events, and are called the **axioms of probability.**

From these axioms we can also derive the following properties satisfied by probabilities:

P_1 : $P(\phi) = 0$. That is, the probability of an impossible event is zero.

P_2 : $P(A \cup B) = P(A) + P(B)$, if $A \cap B = \phi$. That is, if A and B are disjoint, then probability of occurrence of any one of them is the sum of their respective probabilities of occurrence.

P_3 : $A \supset B$ implies that $P(A) \geq P(B)$. If B is a sub-event of A, that is the occurrence of B implies the occurrence of A, then $P(A) \geq P(B)$

P_4 : $A \supset B$ implies that $P(A \sim B) = P(A \cap B') = P(A) - P(B)$.

If B is a sub-event of A, then the probability that A occurs but B does not is the difference $P(A) - P(B)$.

$A \supset B$ means that all the outcomes which are favourable to B are favourable to A too. Thus, if B takes place, then A also takes place.

The above discussion shows that to each event we can assign a number between 0 and 1.

Remark: The probability function is not unique, that is, on the same S and the same set of events $A_1 A_2, \ldots$, many different systems of probabilities can be defined so that the axioms i), ii), iii) are satisfied.

Therefore, in order to define a probability function, it is necessary to know the probabilities of a given set of events. These are, in practice, obtained approximately by repeating a random experiment and determining the proportion of times these events occur.

Uptil now we have defined probabilities of events separately for finite and infinite sample spaces. We shall now prove that the definition of classical probability for finite spaces can be derived as a special case of the definition of the probabilities in an infinite samle space. Let us suppose that the sample space has N sample points, $x_1, x_2, \ldots, x_N$. Then. according to the definition given in Sec. 12.2.1, the probability of each of the events $\{x_i\}$, $i = 1, 2, \ldots, N$, is $\frac{1}{N}$.

Thus, $P(\{x_i\}) = \frac{1}{N}$, $i = 1, 2, \ldots, N$.

You can check that this probability function satisfies the three axioms of probability stated in the beginning of this sub-section. Further, since the axioms of probability are satisfied, we can say that this probability function also satisfies P_1, P_2, P_3, and P_4.

In the next section we'll give some rules which are very useful in evaluating probabilities.

12.3 RULES OF PROBABILITY

We now state some important results which are useful in the evaluation of probabilities of events which can be constructed from the set of events whose probabilities are supposed to be known.

$P(A') = 1 - P(A)$

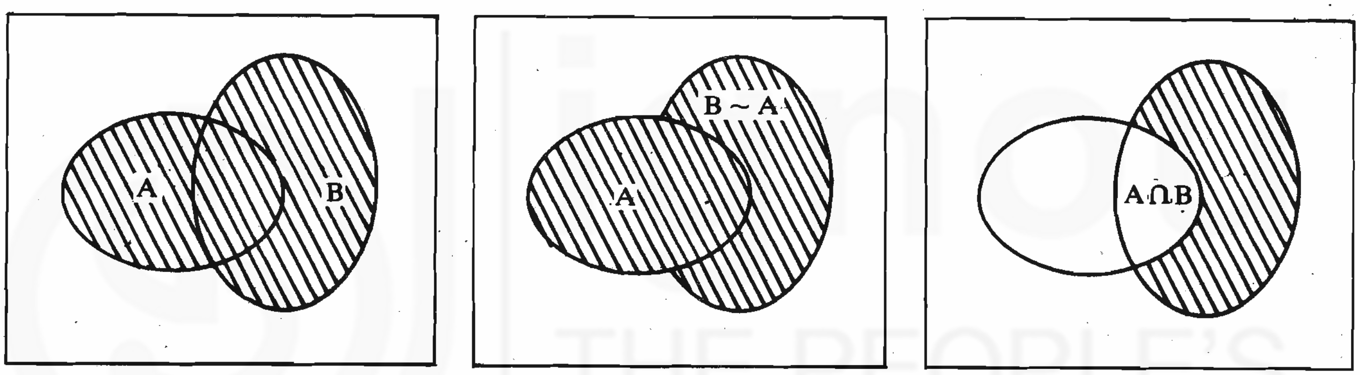
$P(A' \cap B') = 1 - P(A \cup B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

We shall prove only the last result here. The first two are left as an exercise (see E 4)).

Now $A \cup B = A \cup (B \sim A)$, compare Fig. 4(a) and 4(b).

$P(A \cup B) = P(A) + P(B \sim A)$, using P_2.



(a) (b) (c)

Fig. 4

From Fig. 4 you can see that $B \sim A = B \sim (A \cap B)$.

Thus, using P_4, we can write

$P(B \sim (A \cap B)) = P(B) - P(A \cap B)$, and therefore,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

E4) Prove that

a) $P(A') = 1 - P(A)$

b) $P(A' \cap B') = 1 - P(A \cup B)$.

If $A \cap B = \phi$, there are no sample points common to A and B. In other words, there is no outcome which is favourable to both A and B, i.e., $P(A \cap B) = 0$. In such cases , we say that A and B are **mutually exclusive**. Thus, for mutually exclusive events A and B, we get $P(A \cup B) = P(A) + P(B)$.

Let's see how we can compute probabilities using these properties.

Example 2: Of 150 patients examined at a clinic, it was found that 90 had heart trouble, 50 had diabetes and 30 had both. What are the probabilities that a patient, selected at random, had (i) either heart disease or diabetes, (ii) no heart disease but diabetes, (iii) both heart disease and diabetes, and (iv) neither heart disease nor diabetes?

Solution: Let us denote the event that a patient has heart disease by H, and the event that he has diabetes by D. Then H′ and D′ denote the events that the patient is free of heart disease and diabetes, respectively. Here S contains 150 points, each of which represents the outcome of the examination of a patient. The event H includes 90 points, event D, 50 points and event $H \cap D$, 30 points (see Fig. 5), according to the data given.

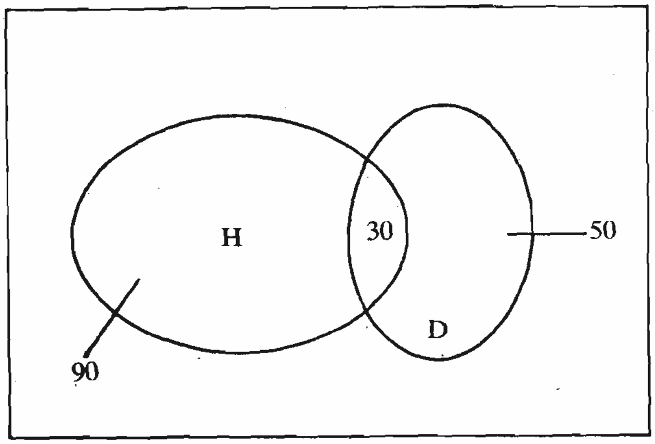


Fig. 5

(i) We have to find $P(H \cup D)$. We know that

$$P(H \cup D) = P(H) + P(D) - P(H \cap D)$$

$$= \frac{90}{150} + \frac{50}{150} - \frac{30}{150}$$

$$= \frac{110}{150} = \frac{11}{15}$$

Recall that in a finite sample space,

$$P(A) = \frac{\text{no. of outcomes favourable to A}}{\text{total no. of outcomes}}$$

(ii) We have to find $P(H' \cap D)$. $(H' \cap D)$ contains 50 - 30 = 20 points, so $P(H' \cap D) = 20/150 = 2/15$.

(iii) We have to find $P(H \cap D)$. Since $H \cap D$ contains 30 points, $P(H \cap D) = 30/150 = 1/5$.

(iv) We have to find $P(H' \cap D')$. Now

$$P(H' \cap D') = 1 - P(H \cup D) = 1 - \frac{11}{15} = \frac{4}{15}.$$

Example 3: A packet contains seeds of two varieties, high yielding and ordinary, numbering 40 and 20 respectively. 10 seeds are picked up from the packet at random. What is the probability that there are 6 seeds of the high yielding variety and 4 ordinary.

An individual is selected at **random** from the population implies that all the individuals in the population had an equal chance of being selected.

Solution: Total no. of points in S = C(60, 10)

Number of points in the event = C(40,6).C(20,4)

Since all points of S have the same probability (see margin remark), the probability of the required event is

$$\frac{C(40,6).\ C(20,4)}{C(60,10)}$$

The required probability $= \frac{6}{36} = \frac{1}{6}$.

Example 4: Given the following frequency distribution of a discrete random variable X, find the probabilities that (i) X has the value 3, (ii) X lies between 2 and 4, both inclusive

X	1	2	3	4	5	6
Frequency	10	20	30	40	50	60

Solution:

Number of points in sample space = 125.
Number of points in event defined by (i) = 30.
∴ Required probability for (i) = 30/125 = 6/25.
Number of points in event defined by (ii) = 90.
∴ Required probability for (ii) = 90/125 = 18/25.

You should be able to do this exercise now.

E5) A river has been polluted because of the industrial waste being dumped into it. The probability that either the fish in the water or the animals that drink from the river will die is 11/21. The probability that the fish will die is 1/3, and the probability that the animals will die is 2/7. What is the probability that both the fish and the animals will die?

12.4 CONDITIONAL PROBABILITY

By now you are familiar with many rules governing the probabilities of events in a sample space. But still there are some situations where we cannot apply these rules. For example, suppose we pick two cards, one by one, from a pack of cards. How can we find the probability that the first card is red, and the second is black? To deal with such situations we shall introduce the concept of conditional probability here. Let us first see what is meant by independent or dependent events.

Suppose two seeds s_1 and s_2 are planted. Suppose s_1 germinates with probability 0.7, and s_2 germinates with probability 0.8. Let S_1 denote the event of s_1 germinating and S_2 denote the event of s_2 germinating. It is obvious that the knowledge that S_1 has taken place is not going to change the probability of occurrence of S_2. Similarly, whether or not S_2 has taken place is not going to influence the occurrence of S_1. Thus, $P(S_2) = 0.8$ whether S_1 has taken place or not; and $P(S_1) = 0.7$ whether S_2 has taken place or not. Such events are called independent events.

Now, let's consider another situation.

A box contains 5 white and 3 red balls. A second identical box contains 3 white and 5 red balls. Suppose we choose a box and draw one ball from it.

Let B : picking the first box. Then B′ : picking the second box.
W : drawing a white ball
R : drawing a red ball.

In this case you will see that if the event B has taken place, that is, if we have selected the first box, then the probability of drawing a white ball is 5/8.

But, if event B′ has taken place, that is, if we have selected the second box, then the probability of drawing a white ball is 3/8.

This means that the probability of the occurrence of W depends on our choice of the box. We express this by saying that the events B and W are dependent events. We also write $P(W|B)$ to denote the probability of W given that B has taken place. Here we have seen that $P(W|B) = 5/8$. We have also seen that $P(W|B') = 3/8$.

Now we give a precise definition of independent events.

Definition 2: Two events A and B are said to be **independent** if the likelihood of the occurrence of event B is in no way affected by the occurrence or non-occurrence of event A.

The probability of event B, given that event A has already occurred, is called the **conditional probability of B given A**, and is denoted by $P(B|A)$. Thus, A and B are independent if and only if $P(B|A) = P(B)$.

You should now be able to do this exercise.

E6) Which of the following pairs of events are independent, and which are dependent?

a) A coin is tossed twice.
A : getting the head in the first toss.
B : getting the head in the second toss.

b) A card is picked from a deck, its colour is noted, and it is then returned to the deck. A second card is picked and its colour is noted.
A : getting a red card in the first pick
B : getting a black card in the second pick

c) A card is picked from a deck and its colour is noted. Without replacing it, a second card is picked and its colour is noted.
A : getting a red card in the first pick.
B : getting a black card in the second pick.

Now we'll see how the conditional probability, $P(A|B)$ can be computed if we know $P(B)$ and $P(A \cap B)$. Suppose the sample space of our experiment has N points. Suppose further, that the number of outcomes favourable to event B is N_B and that of outcomes favourable to event $A \cap B$ is $N_{A \cap B}$. Then

$$P(A \cap B) = \frac{N_{A \cap B}}{N}$$

$$= \frac{N_{A \cap B}}{N_B} \quad \frac{N_B}{N} \qquad \ldots(I)$$

Now what will be $P(A|B)$, the probability of occurrence of A, given B? If B has already taken place, then it means that our sample space is now restricted to B. Now out of these N_B sample points, we will have to find those which are favourable to event A. From Fig. 6, it is clear that these are precisely the points belonging to $A \cap B$.

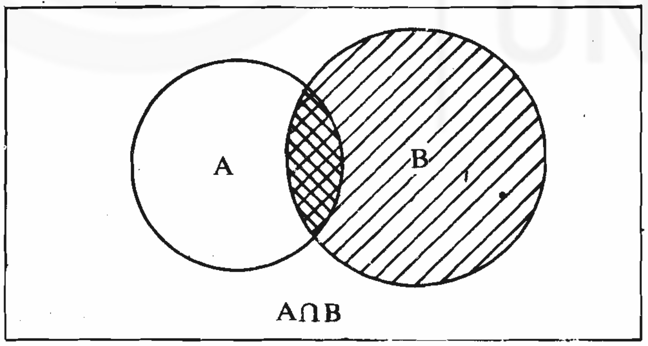


Fig. 6

Therefore, we can say that,

$$P(A|B) = \frac{N_{A \cap B}}{N_B}$$

Hence From (I) we get

$$P(A \cap B) = P(A|B) \cdot P(B)$$

Therefore, we can write $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$, provided $P(B)$ is not zero.

We shall now illustrate the computation of conditional probabilities through an example.

Example 5: It is known that 5 men out of every 100 men and 25 women out of every 10,000 women are colour blind. In a community about half the population is male. (i) What is the probability that a person chosen at random from the community will be colour blind?
(ii) What is the probability that a colour blind person chosen at random from among all colourblind persons in the community will be male?

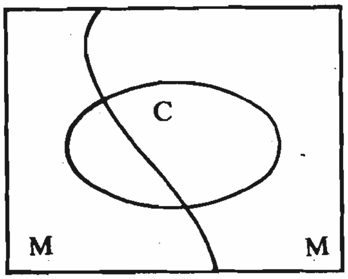


Fig. 7

Solution: Let M denote the event of choosing a male, so M′ denotes the event of choosing, a female, and let C denote the event of being colour blind. Then C′ denotes the event of being normal.

Then from the given data, $P(M) = P(M' = 1/2$,
$P(C|M) = 5/100$, and $P(C|M') = 25/10,000$.
We have to find the probability P(C).

Now, $C = (C \cap M) \cup (C \cap M')$ since a colour blind person can be either male or female. Also see Fig. 7.

Thus, $P(C) = P((C \cap M) \cup (C \cap M')) = P(C \cap M) + P(C \cap M')$, $C \cap M$ and $C \cap M'$ being mutúally exclusive.

$= P(M)\ P(C|M) + P(M')\ P(C|M')$

$$= \frac{1}{2} \times \frac{5}{100} + \frac{1}{2} \times \frac{25}{10000} = \frac{1}{2} \times \frac{525}{10000} = \frac{21}{800}.$$

ii) Now, we shall find the probability $P(M|C)$, probability that the chosen person is male, given that the person is colour blind.

$$P\ (M|C) = \frac{P(M \cap C)}{P(C)} = \frac{1}{2} \times \frac{5}{100} \div \frac{21}{800} = \frac{20}{21}.$$

Thus, we see that the knowledge of occurrence of an event may completely change the probability of occurrence of another. When nothing is known about the state of the eyes (colour blind/normal) of a person, the probability that the person is male is 1/2. But when it is known that the person is colour blind, the probability that the person is male becomes as high as 20/21. This is because the knowledge is very relevant : Colour blindness is much more common among males than among females. So, if there is information that the person is colour blind, it becomes much more likely that he is male.

We shall now turn to Bayes' theorem which is an extension of Example 5.

Bayes' Theorem

The finding that the knowledge of occurrence of one event may substantially change the probability of occurrence of another finds full expression in the following theorem due to Bayes.

Theorem 1 (Bayes' Theorem) : Suppose an event A can occur either in conjunction with H_1, or with $H_2, \ldots,$ or with H_k, where $H_1, \ldots, H_k$ are mutually exclusive. Occurrence of A implies that any one of these events H_i, $i = 1, 2, \ldots, k$ has occurred. Let $P(H_i)$ denote the probability of occurrence of H_i and $P(A|H_i)$ the conditional probability of A, given that H_i has occurred, $i = 1, 2, \ldots, k$. Then $P(H_i|A)$, the conditional probability that H_i has occurred when A is known to have occurred is given by

$$P(H_i|A) = \frac{P(H_i)\ P\ (A|H_i)}{\Sigma\ P(H_i)\ P\ (A|H_i)},\quad i = 1, 2, \ldots\ldots, k,$$

$P(H_i)$ are called the **prior probabilities** of occurrence of H_i, $i = 1, 2, \ldots, k$, and $P(H_i|A)$ the **posterior probabilities** of their occurrence.

If you look back at Example 5, you will find that we had two mutually exclusive events M and M′ in place of $H_1, H_2, \ldots\ldots, H_k$, and event C was the equivalent of event A in Bayes' theorem. The usefulness of this theorem is evident from the following example.

Example 6: A packet contains seeds of 4 grades A, B, C, D. The percentage of the four grades of seeds which germinate are respectively, 80, 50, 40 and 20. The packet contains the four grades of seeds in the proportions 1:2:3:4. A seed is taken from the packet at random, and is planted. If it germinates, what are the probabilities that it was of grade A, B, C, and D?

Solution: Let H_A, H_B, H_C and H_D denote the events that the seed is of grades A, B, C and D, respectively, and G the event that it germinates.

Then $P(H_A) = 1/10$, $P(H_B) = 1/5$, $P(H_C) = 3/10$, $P(H_D) = 2/5$ and $P(G|H_A) = 0.8$, $P(G|H_B) = 0.5$, $P(G|H_C) = 0.4$, $P(G|H_D) = 0.2$, according to the data given.

Now, the probability that a germinated seed is of grade A,

$$P(H_A|G) = \frac{P(H_A)\,P(G|H_A)}{P(H_A)P(G|H_A) + P(H_B)P(G|H_B) + P(H_C)P(G|H_C) + P(H_D)P(G|H_D)}$$

$$= \frac{0.1 \times 0.8}{0.1 \times 0.8 + 0.2 \times 0.5 + 0.3 \times 0.4 + 0.4 \times 0.2}$$

$$= \frac{8}{8+10+12+8} = \frac{4}{19}$$

Similarly, $P(H_B|G) = 5/19$

$P(H_C|G) = 6/19$

$P(H_D|G) = 4/19$

You will be able to solve the following exercises now.

E7) If 4 cards are drawn from a pack of 52, what is the probability that there is one card of each suit?

E8) For the problem in E7), what is the probability that 2 cards are red and 2 black?

E9) Suppose 80% of a particular breed of mice exhibit aggressive behaviour when given a dose of a stimulant. The stimulant is given to 3 mice in succession. Find the probabilities that

a) two or more mice will show aggressive behaviour;

b) the first two will be aggressive, the third normal;

c) exactly two mice will be agressive.

E10) Medical records of male diabetes patients in a hospital for a particular year show the following numbers:

	Light Case		Serious case	
	Diabetes in Parents		Diabetes in Parents	
Age of patient	Yes	No	Yes	No
Below 40	15	10	8	2
Above 40	15	20	20	10

Events A, B, C are defined as follows:

A : Patient is a serious case.

B : Patient is below 40.

C : Parents are diabetic.

Determine the probabilities of A, B, C, $B \cap C$, $A \cap B \cap C$, and explain what these events stand for.

12.5 SUMMARY

In this unit we have

1) introduced the concept of the probability of an event,
2) observed that the probability of any event A, P(A) lies between 0 and 1,
3) noted that Venn diagrams give us a clear idea of various events in a sample space,
4) defined mutually exclusive, dependent and independent events,
5) studied conditional probability and used Bayes' theorem to compute the conditional probabilities of certain events.

12.6 SOLUTIONS AND ANSWERS

E1) a) Rolling the die and noticing the number.
b) The number 3.
c) Six
d) (1, 2, 3, 4, 5, 6)

E2) The outcomes favourable to the given event are (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), and (6, 1), where in (a, b) the first number, a, denotes the number on the first die and the second number, b, denotes the number on the second die. The number of all possible outcomes = 36.

$$\therefore \text{ The required probability } = \frac{6}{36} = \frac{1}{6}.$$

E3) Total number of women promoted during 1984-1988 is 39.
Total number of persons promoted during this period is 98.

$$\therefore \text{ The required probability } = \frac{39}{98} = 0.398$$

E4) a) $1 = P(A \cup A') = P(A) + P(A')$.

$\therefore P(A') = 1 - P(A)$

b) Since $A' \cap B' = (A \cup B)'$

$\therefore P(A' \cap B') = 1 - P(A \cup B)$.

E5) Let A = the event that the fish will die.

B = the event that the animals will die.

Then $P(A \cup B) = \frac{11}{21}$, $P(A) = \frac{1}{3}$, $P(B) = \frac{2}{7}$

We have to find $P(A \cap B)$.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\therefore \frac{11}{21} = \frac{1}{3} + \frac{2}{7} - P(A \cap B)$$

$$\therefore P(A \cap B) = \frac{2}{21}.$$

E6) a) and b): independent
c) dependent

E7) Total number of ways in which 4 cards can be picked from a deck of 52 cards is C (52, 4).

To get 1 card of each suit, we can choose each card in 13-ways. Therefore, the total number of ways in which we can choose 1 card of each suit is $13 \times 13 \times 13 \times 13 = 13^4$.

$\therefore$ The required probability $= \dfrac{13^4}{C(52,4)}$

E8) Number of ways in which we can choose 2 red cards = C(26,2)
Number of ways in which we can choose 2 black cards = C(26,2)

$\therefore$ Number of ways in which we can choose 2 red and 2 black cards = C(26,2). C(26,2).

$\therefore$ The required probability $= \dfrac{C(26,\ 2)\ C(26,\ 2)}{C(52,4)}$

E9) Let M_i, i = 1, 2, 3 denote the event that the ith mouse exhibits aggressive behaviour. Then $P(M_i) = 0.8$, i = 1, 2, 3, and the events are independent.

a) Required probability $= P(M_1 \cap M_2 \cap M_3') + P(M_1 \cap M_2' \cap M_3)$

$+ P(M_1' \cap M_2 \cap M_3) + P(M_1 \cap M_2 \cap M_3)$

$= 0.8 \times 0.8 \times 0.2 + 0.8 \times 0.2 \times 0.8$

$+ 0.2 \times 0.8 \times 0.8 + 0.8 \times 0.8 \times 0.8$

$= 0.896$

b) Required probability $= P(M_1 \cap M_2 \cap M_3') = 0.8 \times 0.8 \times 0.2$
$= 0.128$

c) Required probability $= P(M_1 \cap M_2 \cap M_3') + P(M_1 \cap M_2' \cap M_3')$

$+ P(M_1' \cap M_2 \cap M_3)$

$= 3(0.8)^2\ (0.2) = 0.384.$

E10) $P(A) = \dfrac{8+2+20+10}{100} = \dfrac{40}{100} = 0.4$

$P(B) = \dfrac{15+10+8+2}{100} = 0.35$

$P(C) = \dfrac{15+15+8+20}{100} = 0.58$

$P(B \cap C) = \dfrac{15+8}{100} = 0.23$

$P(A \cap B \cap C) = \dfrac{8}{100} = 0.08$

UNIT 13 DISCRETE PROBABILITY DISTRIBUTIONS

Structure

13.1 INTRODUCTION

Most statistical methods useful to scientists deal with the collection, organisation, analysis and presentation of data. Such analysis of experimental data is used in making reasonable decisions based on the data.

In Unit 11, you have seen that we can organise the data based on a sample result in tabular form with the help of frequency distributions. Using these distributions we can analyse the data. In Unit 12, you studied various rules of probability which enables us to predict how often an event will occur during the entire process. In this unit we combine these ideas and form a **probability distribution.** Probability distribution can theoretically determine the probability of an event depending on the nature of the event and the conditions under which the event is occuring. In this unit you will learn about two such distributions: the **binomial distribution** and the **Poisson distribution.**

You already know from Unit 11 that events are of two types – discrete and continuous. We recall that a discrete random variable can assume values only in a finite set with no possible values located between one value and the next. Whereas, a continuous random variable assumes any one of infinitely large number of values found on a line interval. The probability distribution for discrete variables can be described by binomial and Poisson distributions. These distributions are called **discrete probability distributions.** Here we explain mainly how the binomial and Poisson distributions are generated by taking into consideration the simple laws of probability.

We have also given their means and standard deviations here. In the next unit we shall discuss some probability distributions for continuous variables.

Objectives

After you have completed this unit you should be able to

- describe the characteristics of the binomial and the Poisson distributions;
- choose which distribution to use in a given situation;
- apply the binomial and the Poisson distributions to solve problems;
- obtain their mean and standard deviation.