**BPY-002 LOGIC: CLASSICAL AND SYMBOLIC LOGIC (4 credits)**

**COURSE INTRODUCTION**

Cordial welcome to the BA Philosophy Programme of IGNOU. The curriculum prepared for this degree is relevant and significant. We have included latest scholarship on the course prepared by renowned scholars from across the country.

The second course that you study is "Logic: Classical and Symbolic Logic." The concept of logical form is central to logic; it is being held that the validity of an argument is determined by its logical form, not by its content. Traditional Aristotelian syllogistic logic and modern symbolic logic are examples of *formal logic*. *Informal logic*, which is the study of natural language arguments, includes the study of fallacies too. *Formal logic* is the study of inference with purely formal content. *Symbolic logic* is the study of symbolic abstractions that capture the formal features of logical inference. Symbolic logic is often divided into two: propositional logic and predicate logic. Mathematical logic is an extension of symbolic logic into other areas, such as the study of model theory, proof theory, set theory, and recursion theory. This course presents 4 blocks comprising 16 units.

Block 1 is an introduction to logic. In this block we have tried to explain the nature and scope of logic, concept and term, definition and division, and proposition.

Block 2 deals with meaning and kinds of reasoning, deductive reasoning, syllogisms, and dilemma and fallacies.

Block 3 studies induction, the history and utility of symbolic logic, compound statements and their truth values, and truth-functional forms.

Block 4 probes into formal proof of validity with rules of inference and rules of replacement. This final unit also explains conditional proof and indirect proof, and quantification.

All these four Blocks will enable you to know the core of logic explained in simple terms with profound insights taking into account the latest trends in the discipline. You will have to enhance your knowledge of the subject with the help of the suggested readings and by browsing through the various websites on the topic, especially through the *Wikipedia* on which many contributors depend for their reference.

**BLOCK -1 INTRODUCTION**

Logic is the study of the principles of valid inference and demonstration. The word derives from Greek *logike*, meaning "possessed of reason, intellectual, dialectical, argumentative;" As a formal science, logic investigates and classifies the structure of statements and arguments, both through the study of formal systems of inference and through the study of arguments in natural language. The field of logic ranges from the study of validity, fallacies and paradoxes, to specialized analysis of reasoning using probability and to arguments involving causality. The present block, consisting of 4 units, introduces logic taking into account its basic nature.

Unit 1 on "Nature and Scope of Logic" introduces and familiarizes the nature and scope of the subject to the students. Anyone who ventures to study logic must necessarily know its status among academic disciplines: is it a science or an art? Again, is it a positive science or a normative science? Debates on these cover the problem from various angles. Yet another concern of a student of logic would be the extension and scope of logic. This has been achieved by introspection into the relation between logic and various other sciences.

Unit 2 highlights "Concept and Term." Thought in logic means process or product of thinking concepts. This unit aims at examining major entities in the language of logic like concepts, words and terms. Concept is a word that carries a lot of philosophical significance. Words and terms connote different senses in the language of logic. It is essential that a student of logic gets familiarized with the usage of such technical terms.

Unit 3 explains how Logic deals with thought, and thoughts are always expressed in language. Words we use must convey proper idea. If they have no fixed meaning it would be difficult to understand what one means in his use of a word. In such a situation error and confusion are not uncommon. We define a term according to the interest we have in it. But logic deals with correct thinking. Our thoughts can never be coherent unless we determine the meaning of each term. Now the tools which logic devises for the attainment of this purpose are definition and division.

Unit 4 discusses "Proposition." As we know inferential process is the main subject matter of logic. The term refers to the process by which a proposition is arrived at and affirmed on the basis of one or more other propositions accepted as the starting point of the process. To determine whether an inference is correct the logician examines the propositions that are the initial and end points of that process and the relationships between them. This clearly denotes the significance of propositions in the study of logic.

Four units mentioned above teach us that knowledge is based directly on perception and indirectly on what we have been told. Much of the knowledge we have is justified by means of reasoning or logical argument, based on other knowledge we have. If we are not willing to use reasoning, our beliefs or values might well be inconsistent, and then we would have no knowledge or proper judgement at all.

# UNIT 2 CONCEPT AND TERM

**Contents**



## 2.0 OBJECTIVES

Logic is said to be the study of argument as expressed in language. Language in general is highly ambiguous. In any language words are often used in various senses. For example, sometimes 'thought' and knowledge' are used as synonymous terms. Therefore this unit aims at:

- examining major entities in the language of logic like concepts, words and terms and thereby show that they carry lot of philosophical significance and at the same time carry different senses.
- familiarizing the students with varieties of technical terms. Terms can be classified under different criteria. There are simple and composite terms, singular and general terms, collective and non-collective terms, absolute and relative terms, concrete and abstract terms, positive, negative and private terms and connotative and non-connotative terms. This unit undertakes a study of these various types of terms to procure a good undertaking of the language of logic with which the student is expected to be sufficiently acquainted.

## 2.1 INTRODUCTION

Language is the external expression of intention, thought etc. It is the means of communicating our ideas to other people. In logic, by language we mean only the verbal expression of our ideas, either spoken or written. The Greeks seem to have used the word 'logos' to denote ideas as well as speech. This clearly shows the close relation between language and thought. As Grumbley has rightly pointed out, thought and language are closely connected just like how principles of life and activities of a living organism are connected. Clear thinking and accurate language help each other.

Further, it is language that gives thought a name and an abiding reality as a permanent possession. It is popularly said: 'Thought lives in language'. The multitudes of objects that we see around cannot be remembered unless certain names are endowed to the ideas of those objects. The nature of thought is such that it gets dissolved unless we put them in words.

Language not only structures thought by codifying them, but also does the service of preserving them for future generations. It enables us to split complex ideas into atomic ones to analyse and thereby understand them. Hence the philosophers often comment that logic and language are the two sides of the same coin. In order to understand complex ideas we split them into simple words. Words like *chastity*, *nationality*, *religious* are just a few examples to convince ourselves that words can stand representing condensed expression of complex thoughts pregnant with many ideas. In this unit an attempt is made to understand how words, concepts and terms play a decisive role in our study of logic.

## 2.2 CONCEPT, WORD AND TERM

It is necessary to distinguish between word, concept and term. Concept means a general idea. There is difference between the two ideas represented by the terms 'student' and 'a student'. The term 'a student' refers to a particular student in an indefinite manner and it is essentially singular in usage. The term 'student' is applied in general to all those who undertake studies. The common and essential attributes which are found in every particular individual of the class are thought of separately, and thus we get a concept. In brief, the concept stands for general ideas. When expressed in language concept becomes a term. Judgment is the process of relating two concepts. For instance, the two concepts 'water' and 'cold' may be related and the result is the judgement, 'water is cold'. A judgment when expressed in language is called a proposition.

Sometimes it is said that concepts are mental entities. They are not visible. Conception or simple apprehension is the function of human mind by which an idea of a concept is formed in the mind. It is a process of forming mental image of an object, e.g., you see an elephant and form an idea of the elephant in the mind.

The formation of concepts involves the fallowing processes.

(1) Comparison: Different entities are compared with one another so that the attributes they share in common and those on which they differ can be specified. This process enables the agent to find essential attributes of the concept and distinguish them from what are merely accidental.

(2) Abstraction: The next step is to abstract the essential characteristics. This is purely an intellectual exercise.

(3) Generalization: The third step is to generalize the result of abstraction because obviously not all objects belonging to any given class are observed.

(4) Naming: The final step is to give a name to the generalized group of common attributes, so that it becomes easy to retain the idea of the concept in our mind.

Regarding the nature of conception, there are three views prevalent in metaphysics, Realism, Conceptualism and Nominalism. According to realism there is a corresponding real substance to every concept. This view is attributed to ancient Greek thinkers like Plato. Conceptualism is the view according to which conceptions are not real things but only general ideas. Nominalism is the view according to which conception are merely general names, not general ideas.

What is a word? A word consists of a letter or combination of letters conveying determinate meaning. A word may consist of only one letter. e.g. *a, I*, or it may consist of more than one letter, e.g*., an, man, horse, mortal* etc.

A name is a word or group of words which can become the subject or predicate of a proposition. Every word cannot be called name, e.g., 'or', 'before, 'if' etc. If we say 'Before has four legs' it sounds stupid. Thus it is clear that all words can not become names while all names must be words. Hobbes defines name thus: "A name is a word taken at pleasure to serve for a mark which may raise in our mind a thought like some thought which we had before, and which being pronounced by either, may be to them a sign of what thought the speaker had before his mind". Mill also speaks of two kinds of words: words which are not names (as described above) and words which are names. He calls the latter term.

A term is a word, or a combination of words, which by itself is capable of being used as subject or predicate of a proposition. A proposition is a declarative statement which is either true or false but not both. A term is so called because it occurs at the boundaries of a proposition. In the proposition 'Gandhiji is the father of the nation', 'Gandhiji, and 'father of the nation' are terms because they occur at the boundaries of the proposition. Traditional logic speaks of two kinds of words, viz., subject and predicate. In the example quoted above 'Gandhiji' is the subject because the proposition says something *about 'Gandhiji'* and 'father of the nation' is the predicate because it says something about subject, i. e., 'Gandhiji'. It means that subject term is that about which something is said and the predicate term is what is said *about subject term.* Here 'is' is not a term because it is incapable of functioning either as a subject or as a predicate. Also, names become terms only if they are parts of a proposition as subject and predicate. Thus every word is not term though every term is a word or a combination of words. Again, names may have different meanings, but a term has only one definite meaning in a proposition. Outside the proposition a term loses its significance and is merely a name. For example, *Balance* means a weighing machine or whatever is left after expenditure. However, when we use it in the proposition 'Balance is a weighing machine', it carries only one meaning.

There are three kinds words: *Categorematic*, *syncategorematic* and *a-categorematic*. A categoremtic word is one which can by itself be used as a term without the help of other words, e.g. pencil, clever, man, etc. In other words when a word is used independently either as a subject or a predicate in a statement it is called as categorematic word.
Examples:
Roses are red. (Here 'red' is used as a predicate.)
Red is a color. (Here 'red' is used as a subject.)

All nouns including proper nouns are categorematic words. Let us look at a negative example. Consider a statement; 'roses are very colourful', the word 'very' cannot be independently used. We will not write 'Roses are very', it makes no sense. Nor can we use it as a subject. Again, when we say some are red, we actually mean 'out of many objects only *some* objects are red. Although *some* appears to be a subject it really is not. We understand it by the context in which the statement is made. 'Maradona is a great football player'. Here Maradona is used independently the subject in the statement. In these examples the words 'roses', 'Maradona', 'colourful', etc., are categorematic words.

A syncategorematic word is one which cannot be used independently as term, but which can only be used along with other words e.g., *of, with, and, the,* etc. It is a word that is used as part of a subject or a predicate, or a word that joins the subject and the predicate. Nouns, participles, pronouns, adjectives, etc., are categorematic words, while parts of speech like preposition, adverb, etc., are syncategorematic words. Let us look at a few examples. In the statement 'Some people are funny' 'some' is a part of the subject and so it is a syncategorematic word. The word 'are' joins the subject and predicate, and it is also a syncategorematic word.

Let us look at a few more examples: consider the statement 'Computer is very fast'. Here 'is' and 'very' are parts of the predicate. They are syncategorematic words. In the statement 'the telephone is dead', the word 'the' and 'is' are syncategorematic. Again in the example 'the cat is under the chair', 'under', 'the', and 'is' are syncategorematic words. In fact all words other than nouns and emotive words like Ah! Ouch! Alas, are syncategorematic words. In brief, a categorematic word is one which can be used as a term by itself, without the support of other words and syncategorematic word is one which cannot be used as a term by itself, but can form term only when joined to one or more categorematic words.

A-categorematic words merely express some exclamatory feelings or emotions. Examples: Ouch! Aha! Hurrah! Hymn! Alas! Oh! and similar such exclamations. The word acategorematic may be jaw- breaking, but the words in this classification are pretty easy to identify. It cannot become a term either singly or even when conjoined with other words such as interjection.

This classification of words into three types have been determined by the presupposition that subject predicate form is the basic form and all other sentences or propositions have to be transformed into this form.

At this stage we need to introduce two very important notions: **denotation and connotation**. In the first place terms are used to point out objects, to name and to identify them, e.g., the term 'man' refers to all human beings. When a term is applied to denote objects or show their number, it is said to be used in denotation. It means number, or the reference of a term. As for example, the term 'man' denotes several individuals like Plato Aristotle, Gandhi and others, and all men of past, present and future. Denotation is also known as extension because it shows the extent or range of objects to which a term is applied. All the objects to which a term is extended constitute the extension of a term.

Terms are used not only to denote objects but also to show their qualities. In other words, terms are used to describe the object. Every term has a meaning. It stands for certain qualities. The term 'man' for example, shows the qualities of man like, 'animality' 'rationality' etc. The function of suggesting qualities possessed by this object is known as connotation. Every term denotes certain objects and connotes certain qualities. Connotation is also known as *intension* because it refers to general qualities intended by a term. The extension of the term 'college' is all the various colleges, while its intention is the various qualities describing the term, namely educational institution giving higher education. When we say that connotation of a term consists of the attributes which describe, a question arises as to which attributes are meant by the connotation. There are three views regarding the exact meaning of connotation. 1) *Objective* *view*: according to this view connotation means all the attributes actually possessed by the object, all known and unknown. But since in logic we are not concerned with anything unknown, this view is not useful. 2) *Subjective view*: according to this view we must mention all the qualities known to us. But the subjective position will cause variations regarding the actual qualities of the entity and hence is not acceptable. 3) *Logical view*: according to this view connotation means only those common, essential qualities of the object on account of which the term is applied to the object. Non-essential qualities do not form part of the connotation even if they are common to the whole class.

Let us see a few examples classifying denotation and connotation.

Examples:

| TERM | DENOTES | CONNOTES |
|---|---|---|
| Shoe | All shoes | a stiff outer covering of the foot |
| Knife | All knives | an instrument for cutting |
| Love | No denotation | Fondness, strong liking |

Here are a few more examples of connotation and denotation of terms.

Common noun: 'dog'

Denotation: all the animals to which the term can be applied.

Connotation: a wild or domestic animal of the same genus as the wolf.

Descriptive phrases always have a connotative meaning, but their denotation may be definite, indefinite or totally absent. A definite description can be replaced by a proper noun. Example: 'The proximate island to the south of India' can be replaced by 'Sri Lanka'. An indefinite description can be replaced by a common noun. Examples: 'Baby lion' can be replaced by 'cub'. 'House for a dog' can be replaced by 'kennel'. Naturally if a term does not denote anything (like the term 'love') the question of replacing by common noun does not arise.

## 2.3 TERMS AS A NAME OF CLASS

If we view a term as a name of a class, the connotation of the term defines the essence of that class, while the denotation refers to the members of the class.

Examples: jet, medicine, disease, sports

All these words are terms or classes. Consider one example. Jet is a class of objects. A quality or qualities which make an object jet constitute connotation. All connotative qualities together determine a class. A class may have sub-classes.

Example 1: disease – tropical disease, heart disease, skin disease

While 'disease' is a class it has sub-classes like 'tropical disease', 'heart disease', 'skin disease' etc. These sub-classes may in turn have individual members or further sub-classes. For example, the sub-class, 'tropical disease', has as members 'malaria', 'typhoid', 'cholera' etc. 'Typhoid' is a sub-class having members like 'para-typhoid' etc.

Example 2: Class – 'sports'

'Outdoor sports' is a sub-class of the class 'sports'.

'Cricket' is a member of the sub-class 'outdoor sports'.

The relation of the member 'cricket' to the class 'sports' or to the sub-class 'outdoor sports' is class membership. The relation of the sub-class 'outdoor sports' to the class 'sports' is called class inclusion.

## 2.4 EXTENSION AND INTENSION

It is customary to use 'extension' instead of 'denotation' and 'intension' instead of 'connotation' when a term refers to a class. There is a reason why the words 'extension' and 'intension' are used while we deal with classes. A class may have sub-classes, sub-classes with further sub-classes, and so on as we have seen. By 'extension' we would then mean the range of sub-classes or number of members within that class i.e., how extensive is the denotation of the term, or how wide the denotation of a term is?

Intension means the sum of the qualities which describe a general name. The scope of application of the term is to all the members of the class, and this signifies extension. The qualities or properties of content or subject matter of the term signifies intension.
Let us take the term 'box' as example. The extensional significance of 'box' consists of the objects to which this term may be applied. The intensional significance of the term 'box' is the sum of attributes which defines the class.

## 2.5 INVERSE VARIATION

As the extension increases (covering more sub-classes), the intension decreases and if extension decreases, intension increases. The same relation can be stated in this way also. If intension increases, extension decreases and if intension decreases, extension increases. An example will clarify the relation. Let us employ hypothetical numbers and apply general knowledge. This is enough to understand the nature of relation.

| | | | | |
|---|---|---|---|---|
| Denotation | 3 billion | 1.1 billion | 200 million | 60 million |
| Terms | Asians | Indians | South Indians | Kannadigas |
| Intension | 2 | 3 | 4 | 5 |

A person to be called an Asian must satisfy two requirements; 1) he or she must be a human being 2) that person must be a permanent resident within the geographical boundaries of Asia. Therefore the connotation of 'Asian' is 2. And the population of Asia is approximately 3 billion. Therefore the denotation of the term 'Asians' is 3 billion. 'Indians' constitute a subclass of Asians. Therefore the population of India must be naturally less than that of Asia. An Indian is not only a human living in Asia, but also possesses an additional connotation. He must be a bona fide citizen of India. Therefore the connotation of Indians is one more than that of 'Asians'. The student is advised to try to grasp the rest. It is easy to notice that in this arrangement as denotation decreases connotation increases. If we reverse the arrangement, decrease in intension is accompanied by increase in denotation. The student is advised to experiment to satisfy himself or herself of the truth of this relation.

applicable to null classes.

**Check Your Progress I**

**Note:** a) Use the space provided for your answer.
b) Check your answers with those provided at the end of the unit.

1) Distinguish between word, concept and term.

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2) Explain different classification of words.

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## 2.6 CLASSIFICATION OF TERMS

Terms are classified as simple and composite; Singular and general; Collective and non-collective; Absolute and relative; Concrete and abstract; and, Positive, negative and privative.

**Simple and Composite Terms:**
One worded terms are called simple terms. Examples: Agra, cat, library, etc.
Many worded terms are called composite or complex terms. Examples: highest mountain peak, railway station, group of commandos, spring flowers, Royal Bengal tiger, good humor, wise men of Nottingham etc.

**Singular, General and Collective Terms:**
When a term designates one individual or an object it is called singular term. Examples: Agra, Qutub Minar, etc.
When a term designates many objects or individuals it is called general term. Examples: trees, rivers, men, etc.
A term applicable only to a collection of objects, but not to any individual member, is called a collective term. Examples: library, Indian army etc. The term 'library' is applicable to a set of books, but you cannot pull out a book and call it a 'library'. Similarly the term 'Indian army' refers to a set of soldiers and officers, but we cannot refer to one soldier or officer from the set as 'Indian army'.

**Absolute and Relative Terms:**
A term is an absolute term when its meaning is understood with the help of that term only. Examples: cows, river, etc.
A term is relative when its meaning is understood with the help of some other terms. Examples: grandfather, wife, etc.

**Concrete and Abstract Terms:**

A concrete term refers to objects or a class which exist in space and time and which can be perceived.
Examples: car, Eden garden, stars, fish etc.
An abstract term refers to qualities or entities which cannot be perceived.
Examples: God, demon, love, honesty, virtue, happiness, centaur etc.

**Positive, Negative and Privative Terms:**
A term is positive when it refers to the presence of qualities.
Examples: good, happy, big, train, flowers etc.
A term is negative when it refers to the absence of qualities.
Examples: non-violence, non-cooperation, non-vegetarian etc.
Note that a negative term does not imply an opposite term in the sense of 'black-white', 'hot-cold', 'rich-poor' etc. Prefixes like un-, dis-, as in 'undesirable', 'unbelievable' etc., also do not make a term negative; neither do suffixes like -less, 'powerless', 'homeless' etc., make term negative. It is the meaning that determines its character.

A term is privative when it refers to the deprivation of a quality related to comfort or pleasure. Examples: The term 'deaf' deprives an individual of the 'pleasure of hearing'. 'Pain' deprives one of being painless.

**Check Your Progress II**

**Note:** a) Use the space provided for your answer.
b) Check your answers with those provided at the end of the unit.

1) What do you mean by denotation and connotation of terms?
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2) Write in detail about the classification of terms.
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## 2.7 LET US SUM UP

In this unit we have attempted to understand various linguistic usages with which a student of logic must be familiarized. We started the discussion by distinguishing concepts, words and terms. It was said that a concept is a general idea, while a word consists of a letter or combination of letters conveying some meaning. A term, on the other hand is a word or a combination of words which by itself is capable of being used as subject or predicate of a logical proposition. Logicians use the words extension and intension. Some logicians try to give a mathematical form of expression to the quantitative relation between connotation and denotation. They say, 'connotation and denotation vary in inverse ratio'. Further ,terms can be classified as simple and compose terms, singular and general terms, collective and non-collective terms, absolute and relative terms, concrete and abstract terms, positive, negative and privative, and finally connotative and denotative terms.

## 2.8 KEY WORDS

**Terminology:** Terminology is the study, among other things, of how the terms come to be and their interrelationships within a culture.
**Criterion:** Criterion, in Logic, is an issue or standard used regarding the starting point of an argument or knowledge.

## 2.9 FURTHER READINGS AND REFERENCES

Nath Roy, Bhola. *Text Book of Deductive Logic*. Calcutta: S.C.Sankar & Soul Private Ltd, 1984.
Felice, Anne. *Deduction*. Cochin: 1982.

## 2.10 ANSWERS TO CHECK YOUR PROGRESS

**Check Your Progress I**

1. A concept is a general idea .A word consists of a letter or combination of letters conveying a determinate meaning. A term is a word or combination of words which by itself is capable of being used as subject or predicate of a proposition.

2. Words are classified as categorematic, syncategorematic and a-categorematic words. Categorematic words are those which can by themselves be used as terms without the help of other words. Syncategorematic words are those which cannot be used independently as terms, but which can only be used along with other words, e.g. of, with, and, the, come etc. Acatgegorematic words are words which express only exclamatory feelings or emotions.

**Check Your Progress II**

1. Denotation means number, or the reference of a term. It is also known as extension because it shows the extent or range of objects to which a term is applied. The function of suggesting qualities possessed by this objects is known as connotation.

2. Terms can be classified as:
    Simple and composite terms
    Singular and general terms
    Collective land non-collective terms

Absolute and relative terms
Concrete and abstract terms
Positive, negative and private terms
Connotative and non-connotative terms

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## UNIT 3 DEFINITION AND DIVISION

**Contents**



## 3.0 OBJECTIVES

Logic deals with thought, and thoughts are always expressed in language in which different words we use are expected to convey proper idea. If there are no fixed ideas, it would be difficult to understand what one means by a word. In such a situation error and confusion will be the result. For example, a lawyer and a doctor do not define the term 'man' in the same sense. Their definitions are bound to vary. We define a term according to the interest we have in it. But logic deals with correct thinking. Our thoughts can never be correct unless we determine the meaning of each term through correct language. Each term must be understood in its proper sense. The tools which logic uses to achieve this purpose are definition and division. Therefore, the unit aims at introducing the students to:

- correct thinking
- correct language
- correct knowledge of definition and division

In the previous unit we have seen that a term may be defined in two ways:
1) by reference to its denotation and 2) by reference to its connotation. Explanation of a term is with reference to its denotation and it is known as division, and explanation of a term with reference to its connotation is known as definition. In this unit we undertake a detailed study of definition and division.

## 3.1 INTRODUCTION

Language is a very complicated instrument, the principal tool for human communication. But when words are used carelessly or mistakenly, what was intended to enhance mutual understanding, may, in fact hinder it. Our instrument thus becomes our burden. This can happen when the words used in a discussion are ambiguous or emotionally loaded. True, most controversies involve much more than words, but sometimes conflict turns chiefly on and unsuspected difference in the ways the parties are using some key terms whose different senses may be equally legitimate, but must not be confused. Then it is useful to be able to specify or explain the different senses of the ambiguous term.
Definitions can effectively resolve disputes that are merely verbal by exposing and eliminating ambiguities. Definitions are essential to expose and prevent fallacies of ambiguity and reasoning. We shall begin first by examining the nature of definition.

## 3.2 NATURE OF DEFINITION

Classical logicians have tried to define terms. The term to be defined is called *definiendum* and its definition is called *definien.* According to them, definition aims at unfolding the meaning of a term. It is the explicit statement of complete connotation of a term. The connotation of a term consists of essential attributes of the term. The purpose of defining a term is to understand the meaning of a term. For example, while defining man, *rationality* and *animality* are the two essential qualities which are considered. Hence man is defined as a rational animal. Popularly definition is divided into two types; verbal and real definition.

The time honoured rule of definition is that it is *per genus et differentiam*, i.e., a statement of the connotation of the proximate genus and the differentia of the term. In other words while defining a term one has to state the genus and the differentia. Genus means the class and the differentia means the distinct quality unique to *definiendum* and therefore differentia means *definien.* A definition consists in stating first the class to which the *definiendum* belongs and then state the *definien*. It must be noted that this order is irreversible. In other words, in defining a term, we first of all decide to what class of things it belongs and then, we mark the attribute or group of attributes, which distinguish it from other members of that class. For example, while defining man as a rational animal, it is meant that man possesses the attributes of 'animality', belonging to its proximate genus *animal* and the differentia, *rationality*. It is the differentia because it belongs to none other than man. Similarly, when we define plane triangle as a figure bounded by three straight lines, the proximate genus is figure and the differentium is the attribute of being bounded by three straight lines.

(This view of definition is based on a presupposition that there is a highest class followed by lower classes. The highest class is known as the *summum genus* and the lowest class is known as the *infima species*. Aristotle and the medieval logicians firmly believed that the smaller class is included in the bigger class. This theory of logic of Aristotle is complementary to his theory of biology. Aristotle believed that there are natural classes; genus, species and the entire animal kingdom including the vegetative kingdom can be classified on the basis of genus- species relation. But this type of ordering of terms is not to be found in the domain of language.)

Attributes which we consider in a definition fall into three groups, viz. those which constitute the connotation of a term, those which follow from the connotation (known as properties) and those which neither constitute the connotation nor follow from the connotation (called accidents). If one states the entire connotation, i.e., proximate genus and differentia, we have the definition of the term. If, on the other hand, we enumerate its properties or accidents or merely a part of the connotation we have a *description*. A description is different from definition. While definition states the entire connotation, description states properties, accident and some times a part of the connotation. Definition is scientific while description is popular. The object of the former is to make our ideas of things distinct and clear while the object description is to furnish a rough and ready means of identifying an object.

There are different kinds of definitions

1) **Ostensive definition:** When we explain the meaning of a term by pointing or showing the corresponding object, it is called ostensive definition. For example, when a little child asks what a ball is, the best way to teach him the correct use of this term is to show him a physical object known as ball. Language is not needed to explain the meaning of a term. Thus ostensive definition is non-verbal in nature. All physical objects can be explained in this manner.

2) **Denotative definition:** When a term is defined by referring to the denotation of that term, it is called denotative definition. For example, to know the meaning of the term *Scripture* one can cite the names like the Vedas, the Bible, the Guru Grant Sahib, etc. Such definition is called verbal and denotative. Some times one can make use of the extension of the term to define it. This way of defining term is called extensive definition.

3) **Connotative definition:** When we explicate the connotation of a term, it is called connotational or connotative definition. It explicitly states the connotation of a term. Thus definition should be *per genus et differentia'*, which has been stated earlier.

## 3.3 RULES OF DEFINITION AND FALLACIES

A connotative definition should conform to the following rules;

**Rule I: The definition should state the entire connotation of the term, neither less nor more.**

The connotation of a term consists of common and essential attributes. Therefore, while defining a term we should avoid inessential attributes. Even common attributes may be avoided unless they are at the same time essential attributes as well.

Example: “Man is a rational animal” i.e., Man is that which has animality and rationality. Similarly ‘a plane triangle is figure bounded by three straight lines’.

If this rule is violated, fallacies by stating either *more than* the connotation, or *less than* the connotation are committed. This suggests that the fallacy created by not following Rule I is of *two types*. Let us examine each separately.

A. If the definition states more than the connotation, the additional attribute would be either 1) superfluous or 2) an inseparable accident or 3) a separable accident, leading to the fallacies of *Redundant*, *Accidental* and *Too Narrow definitions*.

1. **Fallacy of Redundant definition:** If the additional attribute be a property we have the fallacy of redundant definition. The additional attribute is a common attribute but not an essential attribute. Hence it is superfluous to state it in a definition. For example, the definitions of triangle as “A plane figure, bounded by three straight lines, and having three angles” is a redundant definition because, the attribute of “having three angles” is superfluous.

2. **Fallacy of Accidental definition:** If the additional attribute be an inseparable accident, we have the fallacy of accidental definition. For example, the definition of man as “A laughing rational animal “ is an accidental definition, because the attribute laughing even though as an attribute is found at times in men, is not a part of the connotation of the term man.

**3. Fallacy of Too narrow definitions:** If the additional attribute be a separable accident we have the fallacy of too narrow definition, because it is no longer applicable to its whole term but only to a part of it. For example the definition “Man is a civilized rational animal” is too narrow as the attribute *civilized* does not belong to all men. Similarly, if we were to define a triangle “as a plane figure enclosed by three equal straight lines”, it is not sufficiently extensive.

B. Now let us attend to the next section. If the definition states something *less than* the connotation we have the fallacy of too wide definition. It is so called because it will apply to a greater number of things than are included in the denotation of the term defined. For example, “diamond is carbon” is too wide because it not only applies to diamond but also to all things made up of carbon.

**Rule II: The definition should be clearer than the term defined and should not, therefore, be expressed in figurative, ambiguous or obscure language.**

Violation of this leads to the fallacies of *figurative* and *obscure* definitions.

Examples for **figurative definitions**: 1) “Childhood is the morning of life”

2) “Necessity is the mother of invention”

Example for **obscure definitions:** A girl is a perpendicular biological phenomenon in short skirt.

**Rule III: The definition should not contain the term defined, or a synonym of it.**

Violation of this rule leads to the fallacy of *circular definition*. For example “the sun is the center of the solar system”. Here the term *solar system* already presupposes *Sun* that is to be defined.

**Rule IV**: **A definition should not be negative when it can be affirmative. A definition should positively state what the term means if it is possible to make an affirmation about it. A negative proposition merely states what a term does not mean.**

Violation of this rule leads to the fallacy of negative definition.

Examples: 1) "Mind is not matter."

2) "Failure is but want of success."

When we find it difficult or absolutely impossible to define a term, the so-called negative definition may come to one's aid to describe the entity. In Indian philosophical tradition while defining Brahman, the Advaita resorts to this type of definition presenting well the incapability to connote Brahman positively. Indian logicians however took objections to this type of definition.

To conclude, a definition should be a precise, clear and adequate, and should not be tautologous, redundant or negative.

## 3.4 LIMITS OF DEFINITION

Following are the limits of definition:

*Summum genus* cannot be defined. We have already seen that a definition should be *per genus et differentiam*. The *summum genus*, being the highest genus, cannot be brought under a still higher genus and there fore, it cannot be defined.

Singular abstract names, which are names of elementary attributes, cannot be defined because there is nothing simpler or more elementary than what they are. For example, terms like equality, energy, etc. cannot be defined.

Proper names and individual objects are indefinable. Proper names cannot be defined since they do not possess any connotation. Individual objects posses an infinity of attributes and therefore it is impossible to complete enumeration of all the attributes of them. Hence they too cannot be defined.

**Check Your Progress I**

**Note:** a) Use the space provided for your answer.
b) Check your answers with those provided at the end of the unit.

1) What is definition and what are its different kinds?
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2) Explicate the rules of connotative definition.
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## 3.5 ON DIVISION

Aristotle and the medieval logicians tried to integrate definition with division. For them to define means to divide and vice versa. According to these logicians definition also means the division of bigger classes into proximate classes. Only classes are divided, individuals cannot be divided. Also the smallest class cannot be divided at all. The smallest or lowest class is known as the *infima species*. It should be remembered in this connection that Aristotle and the medieval logicians conceived language as consisting of only classes and sub-classes. But as a matter of fact this is not so. There are various types of words and terms in language which do not fit into this scheme. Let us understand more about Division.

Division is the splitting up of genus or higher class into its constituent species or subclasses according to a certain principle. It is different from definition to the effect that the former is the analysis of the denotation of a term while the latter is the statement of its connotation. In fact logical division is division of a class into sub-classes and not a division of an individual thing into its different parts. To this extent it is different from natural division.

There are various types of division viz., 1) natural division, 2) metaphysical division and 3) logical division. Classification and division which characterize biology is an example of natural division because it is easily discerned in nature itself. Man has no role to play in it. Metaphysical division is, on the other hand, the same as conceptual analysis. Substance- attribute, cause-effect, space-time, particular- universal, etc., illustrate metaphysical division.

Both natural division and metaphysical division should be distinguished from logical division. Unlike the former two types it cannot be applied to an individual thing but only to a class of things. Logical division is the analysis of the extension of class terms. Here one splits a genus into its constituent species. It is closely connected with the process of classification of connotative definition. It is said that in defining we divide and in dividing we define. In order to define the term *man*, we state its genus *animal* and its differentia *rational*. This necessarily implies that the class of animal can be divided into two sub-classes from the standpoint of having or not having rationality, i.e., man and not-man. This way of defining involves division. Again, when we divide triangle into *equilateral, isosceles and scalene* taking into consideration the equality of sides, the definitions of these terms are evident, since their genus is triangle and the differentia are *having three equal sides, having two equal sides and having unequal sides* respectively. Thus division involves definition. When the term animal is divided, the term man is defined and when the term man is defined, the term animal is divided. Thus the primary aim of division is to make the meaning of the term clear.

## 3.6 RULES OF LOGICAL DIVISION

Logical division should abide by the following rules that follow from the very nature of the division**.**

**Rule I: The term to be divided must be a general term:**

This rule is evident from the very definition of logical division. It is only a class, which can be divided into its sub-classes. Division aims at giving us a complete idea of the extension of the term and all the sub classes constitute the extension of the class.

**Rule II: Logical division must be according to one definite principle:**

If more than one principle is adopted we shall commit the fallacy of cross division. Division of students into tall, intelligent, fair and backbenchers is a case of cross division. Here the sub classes get mixed up together. In this case we have adopted four principles of division, namely intelligence, height, complexion and sitting habit. Consequently the very purpose of division is defeated.

**Rule III: The name of the class divided must be applicable to each of the subdivisions coming under it:**

All subclasses of a higher class belong to that class. Hence in every logical division the sub-divisions may take the name of the class. Thus when the term man is divided into the subclasses, tall, short and medium sized, all these subdivisions being subdivisions of the class *man*, we can tell them to be tall man, short man and medium sized man. But division of man into head, hands, legs etc. is not a case of logical division. In these cases it is not possible to apply the term to each of the above parts, the 'head' is not man, 'hands' are not man.

**Rule IV: The sub-classes taken together exhaust the extension of the term defined:**

Division aims at giving us a complete idea of the extension of the term. Denotational definition is bound to be incomplete and hence extensional definition is preferred. In giving extensional definition we point out all the subclasses and if any sub-class is left out the division is incomplete. Dividing triangle into acute angled and right angled is incomplete because obtuse angle triangles are left out.

**Rule V: The sub-classes to which the term is divided must be mutually exclusive.**

This follows from the rules that division must be always on single and fundamental principle. If the classes are not mutually exclusive we are sure that more than one principle have been adopted and the second rule has been violated. Thus the division of man into rich, tall and honest illustrates the fallacy of overlapping division. The subclasses are overlapping, not exclusive.

**Rule VI: In a continued division each step should divide a class or sub-class into its proximate sub-classes**.

This means division must not take a leap. If a logical division involves more than one step, it should be continuous, proceeding step by step without omitting any intermediate species. Violation of this rule leads to the fallacy of too narrow division. For example, rectilinear plane figures should not be divided immediately into such remote species as equilateral triangles, squares, parallelograms etc.

It may also be noted that the rules mentioned above are all inter- connected. Hence the violation of any one of them may, at the same time, involve violation of other rules as well.

## 3.7 DIVISION BY DICHOTOMY

In many cases, it is difficult to assure us whether all the rules have been duly satisfied or not. Further, without material knowledge of the things denoted by the term it is not possible to have a correct form of logical division. In order to avoid these difficulties, a form of division called *Division by dichotomy* is suggested. Dichotomy literally means dividing into two. Division by dichotomy is illustrated when we divide a class into two complementary subclasses. For example, if we divide people of the world into Asians and non-Asians, then we have division by dichotomy. For someone familiarized with the rules of division it is clear that to assume ourselves whether all the rules have been duly satisfied or not seems an uphill task. Further, without the material knowledge of things denoted by a term, it is not possible to have a correct form of logical division. Since there is more than one principle of division, subclasses must not overlap and when taken together the subclasses should be equal to the class divided. Now it is clear that we are incapable of being certain that a particular logical division conforms to all the rules if we lack knowledge of the things denoted by the class to be divided. This kind of material knowledge is wanting in formal logic. Hence formal logicians conceived this kind of division.

This division is done by mere form of the division. In this type even without my knowledge of the subject matter, which is being divided, we may be certain that the rules of division have been observed. Such a type of division is suggested to avoid difficulties that may arise as cited above (in fact some logicians consider division as a part of material logic).

There cannot be more than one principle of division operating simultaneously. Therefore two subclasses can be obtained according to the principles of excluded middle and non – contradiction and therefore they must be mutually exclusive and together must be equal to the denotation of the class divided. In this way, the rules of division are observed, yet knowledge of the subject matter is not necessary.

Division by dichotomy has its strength and weakness. Its strength is that it ensures the completeness of a division in a formally perfect fashion as it is based on the laws of contradiction and excluded middle. At the same time, it is open to the serious objection that this type of division is superficial whereas what is expected of logical analysis is much deeper and clear division.

**Check Your Progress II**

**Note:** a) Use the space provided for your answer.
b) Check your answers with those provided at the end of the unit.

1) What is division? Explain various kinds of division?

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2) Sate and explain rules of logical division.
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## 3.8 LET US SUM UP

In this unit we have taken up a very important problem in logic, namely the nature, types, functions and fallacies of definition along with logical division, which should necessarily form part of any course in logic. The problem of definition is clubbed with division since the course to be studied along with definition carries almost the same subject matter and their explanations are mutually dependent.

We have seen that definition is the explicit statement of all the essential attributes connoted by a term. The purpose of defining a term, it was clarified, is to understand the nature of a term. After examining the nature of definition we have looked into the various rules of definition, violation of which would end up with definitional fallacies. It was noted that certain entities or terms are beyond the scope of definition and therefore, remain indefinable. Definition and division are interconnected issues. Different types of division viz., physical, metaphysical and logical were also discussed. Of these it was logical division that demands the attention of logicians. In division there are six rules, violation of which leads to fallacies of division. Since in many cases it is difficult to assure ourselves whether all the rules have been duly satisfied or not, logicians propose a type of division applicable in formal logic, namely division by dichotomy. Division by dichotomy is that type of division, which divides a class into two contradictory sub-classes, for example, the class of people on earth into Asians and not-Asians.

## 3.9 KEY WORDS

**Meaning:** Meaning is associated with connotation. It is precisely what we ought to understand.
**Language:** Language is the systematic creation and usage of systems of symbols referring to linguistic concepts with semantic or logical or otherwise expressive meanings.
**Predicables**: Predicables are the possible relations of the predicate to the subject. In this regard logician Porphyry spoke of five predicables, viz., genus, species, differentia, property and accidents. Genus and species refer to the denotative function of the terms; the other three refer to the connotative functions.

## 3.10 FURTHER READINGS AND REFERENCES

Copi, Irving M. and Cohen, Carl. *Introduction to Logic*. New Delhi: Prentice-hall of India Private Limited, 1997

Felice, Anne. *Deduction*. Coclin , 1982

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## 3.11 ANSWERS TO CHECK YOUR PROGRESS

**Check Your Progress I**

1. The connotation of a term consists of common and essential attributes included in the term and definition means an entire connotation of the term. The purpose of definition is to understand the nature of the term. There are different kinds of definition: ostensive, denotative and connotative definitions. Ostensive: defining by pointing to the object; denotative: definition by referring; connotative: defining per genus et differentia.

2. Rule 1: A definition should state the entire connotation of the term, neither less nor more.

   Rule 2: A definition should be clearer than definiendum and should not, therefore, be expressed in figurative, ambiguous or obscure language.

   Rule 3: A definition should not contain definiendum or a synonym of it.

   Rule 4: A definition should not be negative when it can be affirmative.

**Check Your Progress II**

1. Division is the splitting up of genus or higher class into its constituent species or subclasses according to a certain principle. These are various kinds of division: natural, metaphysical and

logical. Natural: division among living beings, Metaphysical: conceptual analysis undertaken by philosophers, Logical: the analysis of the extension of class term.

2. Rule 1: The term to be divided must be a general term.
   Rule 2: Division must be according to one definite principle.
   Rule 3: The name of the class divided must be applicable to each of the Sub-divisions coming under it.
   Rule 4: The subclasses taken together exhaust the extension of the term defined.
   Rule 5: The subclasses to which the term is divided must be mutually exclusive.
   Rule 6: In continued division each step should divide a class or subclasses into its proximate sub-classes.

## UNIT 4 PROPOSITIONS

**Contents**



## 4.0 OBJECTIVES

As we know inference is the main subject matter of logic. The term refers to the argument in which a proposition is arrived at and affirmed or denied on the basis of one or more other propositions accepted as the starting point of the process. To determine whether or not an inference is correct the logician examines the propositions that are the initial and end points of that argument and the relationships between them. This clearly denotes the significance of propositions in the study of logic. In this unit you are expected to study:

- the nature
- the definition
- the types and forms of propositions
- the difference between propositions and sentences and judgments
- the description of various types of propositions viewed from different standpoints like, composition, generality, relation, quantity, quality, and modality.

## 4.1 INTRODUCTION

Classical logic concerns itself with forms and classifications of propositions. We shall begin with the standard definition of proposition. A proposition is a declarative sentence which is either true or false but not both. Also a proposition cannot be neither true nor false. A proposition is always expressed with the help of a sentence. For example - the same proposition “It is raining” can be expressed in English, Hindi, and Sanskrit and so on. It means that two or more than two sentences may express the same proposition. This is possible only when proposition is taken as the meaning of the sentence which expresses it. Therefore sentence is only the vehicle of or the means of expressing a proposition. It is the unit of thought and logic whereas sentence is the unit of grammar. A sentence may be correct or incorrect; the grammatical rules determine this. A proposition may be true or false, the empirical facts determine the status. The primary thing about a sentence is its grammatical form, but the primary thing about a proposition is its meaning and implication.

The different types of sentences are not different types of propositions. Some types of sentences are not propositions at all. Sentences may be assertive, interrogative, and imperative. Only assertive types of sentences are propositions and rest of them are not (for more details, see below 4.3).

A set of proposition make up an argument. Let us see what role propositions play and how logicians will be concerned in logic by taking a simple example of an argument:

| | |
|---|---|
| All men are mortal. | proposition1 |
| All kings are men. | proposition2 |
| Therefore all kings are mortal. | proposition3 |

Given these propositions as true or false, the logician will only find out whether the argument is valid or not by using certain rules that we shall learn later. Before we proceed further, it is of importance that we situate the discussion on "Proposition" in the whole context of the history of Logic itself.

## 4.2 HISTORY OF LOGIC AND PROPOSITION

Aristotle, the classical logician defines proposition as that which contains subject, predicate and a copula. "Rose is red" is a proposition. Here 'Rose' is the subject, 'red' is the predicate and 'is' is the copula. A subject is that about which something is said, a predicate is what is said about the subject and the copula is the link. Further, according to classical logicians copula should be expressed in the form of present tense only. That is why classical logicians talk of reduction of sentences into propositions. According to them all propositions are sentences but all sentences are not propositions. Subject-predicate logic ultimately gave rise to substance-attribute metaphysics in philosophy.

Aristotle classifies proposition into four types. They are as follows: Universal affirmative (A); Universal negative (E); Particular affirmative (I) and Particular negative (O). These propositions are called categorical or unconditional propositions because no condition is stated anywhere in the propositions. Letters within parentheses are standard symbols of respective propositions which are extensively used throughout our study of logic. "All men are mortal" is an example of 'A' proposition. "No men are immortal" is an instance of 'E' proposition. "Some men are intelligent" is an 'I' proposition and "Some men are honest" is an instance of 'O' proposition.

Aristotle was the first thinker to devise a logical system. He holds that a proposition is a complex involving two terms, a subject and a predicate. The logical form of a proposition is determined by its quantity (universal or particular) and quality (affirmative or negative). The analysis of logical form, types of inference, etc. constitute the subject matter of logic.

Aristotle may also be credited with the formulation of several metalogical propositions, most notably the Law of Noncontradiction, the Principle of the Excluded Middle, and the Law of

Bivalence. These are important in his discussion of modal logic and tense logic. Aristotle referred to certain principles of propositional logic and to reasoning involving hypothetical propositions. He also formulated nonformal logical theories, techniques and strategies for devising arguments (in the Topics), and a theory of fallacies (in the Sophistical Refutations). Aristotle's pupils Eudemus and Theophrastus modified and developed Aristotelian logic in several ways.

The next major innovations in logic are due to the Stoic school. They developed an alternative account of the syllogism, and, in the course of so doing, elaborated a full propositional logic which complements Aristotelian logic. They also investigated various logical antinomies, including the Liar Paradox. The leading logician of this school was Chrysippus, credited with over a hundred works in logic. There were few developments in logic in the succeeding periods, other than a number of handbooks, summaries, translations, and commentaries, usually in a simplified and combined form. The more influential authors include Cicero, Porphyry, and Boethius in the later Roman Empire; the Byzantine scholiast Philoponous; and alFarabi, Avicenna, and Averroes in the Arab world.

The next major logician of proposition is Peter Abelard, who worked in the early twelfth century. He composed an independent treatise on logic, the Dialectica, and wrote extensive commentaries. There are discussions of conversion, opposition, quantity, quality, tense logic, a reduction of de dicto to de re modality, and much else. Abelard also clearly formulates several semantic principles. Abelard is responsible for the clear formulation of a pair of relevant criteria for logical consequences. The failure of his criteria led later logicians to reject relevance implication and to endorse material implication.

Spurred by Abelard's teachings and problems he proposed, and by further translations, other logicians began to grasp the details of Aristotle's texts. The result, coming to fruition in the middle of the thirteenth century, was the first phase of supposition theory, an elaborate doctrine about the reference of terms in various propositional contexts. Its development is preserved in handbooks by Peter of Spain, Lambert of Auxerre, and William of Sherwood. The theory of obligations, a part of non-formal logic, was also invented at this time. Other topics, such as the relation between time and modality, the conventionality of semantics, and the theory of truth, were investigated.

The fourteenth century is the apex of mediæval logical theory, containng an explosion of creative work. Supposition theory is developed extensively in its second phase by logicians such as William of Ockham, Jean Buridan, Gregory of Rimini, and Albert of Saxony. Buridan also elaborates a full theory of consequences, a cross between entailments and inference rules. From explicit semantic principles, Buridan constructs a detailed and extensive investigation of syllogistic, and offers completeness proofs.

## 4.3 PROPOSITIONS AND SENTENCES

Propositions are stated using sentences. However, all sentences are not propositions. Let's look at a few examples of sentences:

1. Snakes are poisonous.
2. Some students are intelligent.
3. How old are you?
4. May God bless you!
5. What a car!
6. Vote for me.

The first two statements are assertions and we can say of these statements that they may either be true or false. Therefore they are propositions.

However, we cannot say whether or not the question, 'How old are you?' is true or false. The answer to the question, 'I am 16 years old' may be true or false. The question is not a proposition, while the answer is a proposition.

'May God bless you' is a ceremonial statement and it is neither true nor false. Therefore, such statements are not propositions.

'What a car!' is exclamatory and has nothing to do with being true or false. Exclamatory statements are not propositions.

'Vote for me' is an appeal or command. We cannot attribute truth or falsity to it. Therefore, evocative statements are not propositions.

We therefore need to distinguish between sentences and propositions. The differences are:

1. Propositions must be meaningful (meaningful in logical sense) sentences.
2. Propositions must have a subject, a predicate and a word joining the two, a sentence need not.
3. All propositions are either true or false, but sentences may or may not be.
4. Propositions are units of Logic, sentences are units of Grammar.

## 4.4 PROPOSITIONS AND JUDGMENTS

Till the nineteenth century, idealistic philosophers used the word, 'Judgment' instead of 'propositions'. Nowadays, a distinction is made between the two words. "Judgment" means 'pronouncing a formal decision'. "Proposition" means 'the result of judging'. Judgment is basically the attitude we take whereas proposition is that which we affirm or deny, accept or reject as true or false. Judgment is a mental act, a process, and an event in time. Proposition is time invariant.

When we say 'All kings are mortal', it is a proposition. When we assert 'We believe that all kings are mortal', we are in fact taking an attitude, making a judgment. Sometimes, a statement may appear by itself to be a proposition. However, if one knows the context in which the statement is made, it may turn out that the proposition is really a judgment made.

Consider the statement: 'All foreigners are unacceptable'. By itself, it looks like a proposition, but what, if a speech is made and at the end the speaker concludes logically why 'all foreigners are unacceptable'. In such a case the speaker is actually passing a judgment. Sometimes, therefore, we need the context to distinguish a proposition from a judgment.

It is only in the beginning of twentieth century that A.N. Whitehead and Bertrand Russell recognize varieties of propositions. According to them subject-predicate logic is only one form of propositions.

## 4.5. TYPES OF PROPOSITION

Propositions can be viewed from different standpoints and classified into different types:

| STANDPOINT | TYPES OF PROPOSITIONS |
|---|---|
| Composition | Simple, Complex or Compound |
| Generality | Singular, General |
| Relation | Categorical, Conditional |
| Quantity | Universal, Particular |
| Quality | Affirmative, Negative |
| Modality | Necessary, Assertoric, Problematic |
| Significance | Verbal, Real |

**Composition - Simple Propositions**

Examples: Love is happiness.
Tiger is ferocious.
All white men were dreaded by the red Indians.

A simple proposition has only one subject and one predicate. Note that the subject 'All white men' is one subject though it has many words. Similarly 'Red Indians' is one predicate.

**Composition – Complex or Composite Propositions**

Examples: Violence does not pay and leads to unhappiness.
She is graceful but cannot act.
Either he is honest or dishonest.
If John comes home, then you must cook chicken.

'She is graceful' is a simple proposition. 'Cannot act' can be written as 'She cannot act', which is a simple proposition again. These simple propositions are connected by a conjunction 'but'. When two or more simple propositions are combined into a single statement we get a complex or composite proposition.

**Generality: Singular proposition**

Examples: The dog wags its tail.
George is my friend.

Kapil Dev is a good cricketer.

When in a proposition the subject refers to a definite, single object, the proposition is said to be singular proposition. A proper noun or a common noun preceded by a definite article 'the' forms the subject of such a proposition.

**Generality - General Propositions**

Examples: Children like chocolate.
All hill stations are health resorts.
Some people are funny.
Few bikes come with fancy fittings.

When in a proposition the subject refers to many objects, the proposition is said to be a general proposition. A common noun forms the subject of such propositions. When it is singular, the indefinite article 'a' is used. 'A dog' means any dog. It generalizes across all dogs. Words like 'some', 'few' refer to more than one object.

**Relation - Categorical Propositions**

Examples: The pillows are soft
Junk food is not good for health
Music is the food of love.

A proposition that affirms or denies something without any condition is called a categorical proposition. Recall that a proposition has a subject, a predicate and a joining word. The joining word relates the two together. In the first example the subject, "the pillows" is joined to the predicate "soft" by the joining word "are'. In this proposition the softness of the pillow is asserted or affirmed. In the second example it is denied that junk food is good for health.

Simple and general propositions are categorical in nature. In the above examples there are no conditions relating the subject and the predicate. Therefore they are called categorical propositions.

**Relation: Conditional Propositions**

Examples: If you study hard, then you will do well.
Robert is either an athlete or a carpenter.

A conditional proposition consists of two categorical propositions that are so related to each other that one imposes a condition that must be fulfilled if what the other asserts is to be acceptable.

There are three types of conditional propositions:

1. Hypothetical proposition

2. Alternative proposition
3. Disjunctive proposition

**1. Hypothetical Proposition**

Examples: If (you are hungry), then (you can eat chocolates.)
If (it doesn't rain), then (the harvest will be poor.)

A hypothetical proposition consists of two categorical propositions. They are put within parentheses. The first part is called antecedent and the second part is called consequent. These two propositions are related in such a way that if the first is true then the second must be true if the second is false, then the first also is false. However, if the first part is false, the second part may be true or may be false.

Example: If the sun shines then there is light
---------------- ----------------
antecedent consequent

**2. Alternative Proposition**

Examples: John is either a professor or a musician
Either we play football or we play cricket
John is either a doctor or the author of this book.

An alternative proposition consists of two simple categorical proposition connected by 'either – or' and thus suggesting that any one of these two proposition may be true or both may be true. John may be a professor or may be a musician. It is also likely that John is both a professor and a musician. The two parts of an alterative proposition are known as alternant. Either alternant may be true or both may be true. The alternative proposition will be false only when both the alternant are false (see for details block 3, 2.3).

| ***Either (Alternant)*** | ***Or (Alternant)*** | ***Proposition*** |
|---|---|---|
| John is a professor | John is a musician | |
| TRUE | TRUE | TRUE |
| TRUE | FALSE | TRUE |
| FALSE | TRUE | TRUE |
| FALSE | FALSE | FALSE |

**3. Disjunctive Proposition**

Examples: It is not the case that both he is honest and he is dishonest.
It is not the case that both the meat is boiled and roasted

A disjunctive proposition consists of two simple categorical propositions (alternant) which are so related that both cannot be simultaneously true.

Note: The fact that both cannot be true at the same time is the only difference between an alternative and disjunctive proposition. Thus there may be examples which are common to both. In symbolic logic we use disjunctive for alternative and the third variety is called negation.

Examples: Either he is in the class or he is in the playground.

| ***Either (Alternant)*** | ***Or (Alternant)*** | ***Proposition*** |
|---|---|---|
| | John is in class | John is in playground |
| FALSE | TRUE | TRUE |
| TRUE | FALSE | TRUE |

**Modality: Assertoric Proposition:**

Examples: The earth moves round the sun.
Objects far away appear small to the eyes.
At zero degree centigrade water turns into ice.
Eleven players form a cricket team.
The earth is not perfectly round.

When the claim or assertion made in a proposition is verifiable it is called an assertoric proposition. The assertion that the earth moves round the sun can be verified by scientific methods. If the result of such verification is true then the proposition is true.

**Modality: Necessary Proposition:**

Examples: Bachelors are unmarried male.
The result of any number multiplied by zero is zero.
A point has no dimension.

Propositions which are always true by definition are called necessary propositions.

**Modality: Problematic Proposition:**

Examples: Perhaps he is a rich man.
She may be happier off with him.
There may be famine this year.

In a problematic proposition we only guess the truth or falsity and make no definite assertion.

**Quantity - Universal Proposition:**

Examples: All boys in the team are educated.
No politicians are honest.
Shillong is a hill station.

When the predicate tells something about the entire class referred to by the subject term, it is called a universal proposition. The predicate term 'educated' refers to the entire class referred to by the subject term 'all boys in the team'.

**Quantity - Particular Proposition:**

Examples: Some girls are beautiful.
Some songs are classical.
Some men are religious.

When the predicate term tells something about an indefinite part of the class referred to by the subject term, it is called particular proposition.

**Quality:**

The early discussion on proposition from the standpoint of quantity was based on the subject class being quantified by the word all, some, no etc. When we discuss proposition from the standpoint of quality our focus will be on the 'copula' between the terms. A copula relates the two terms and is of some form of the verb 'to be'- 'is', 'are', 'is not', 'are not'

The copula either affirms or denies the relation between two terms

**Quality: Affirmative Proposition**

Examples: Some fruits are sweet.
All computers are fast.
Mr.John is bald.

If the relation between the subject term and the predicate term is positive (or affirmative), the proposition is said to be affirmative. In this case the copula is of the form 'is' or 'are'.

**Quality: Negative Proposition:**

Examples: Some fruits are not sweet.
All computers are not fast.
Mr. John is not bald.

If the relation between the subject term and the predicate term is negative (or denied), the proposition is said to be negative. In this case the copula is of the form 'is not' or 'are not'

## 4.6 QUALITY AND QUANTITY

So far we have viewed a proposition from various standpoints like composition, relation, modality and so on. More important of these are the standpoints of quality and quantity in viewing categorical propositions. Recall that:

Quantity: Universal
Particular

Quality: Affirmative
Negative

If we view a proposition from a combined stand point of quality and quantity, we get the following classification as in Aristotle's logic:

| Quality | Classification | Forms of Proposition |
|---|---|---|
| 1. Universal+ Affirmative | A | All (…) are/is (…) |
| 2. Universal+ Negative | E | No (…) are/is (…) |
| 3. Particular+ Affirmative | I | Some (…) are (…) |
| 4. Particular+ Negative | O | Some (..) are not (..) |

**Check Your Progress I**

**Note:** a) Use the space provided for your answer.
b) Check your answers with those provided at the end of the unit.

1) What is a proposition? Distinguish it from sentence.
……………………………………………………………………………………………
……………………………………………………………………………………………
……………………………………………………………………………………………
……………………………………………………………………………………………

2) Mention Aristotelian classification of proposition.
……………………………………………………………………………………………
……………………………………………………………………………………………
……………………………………………………………………………………………
……………………………………………………………………………………………

## 4.7 LET US SUM UP

In the above unit we have seen how important it is to reduce sentences to its logical form, namely propositions. However, while changing sentences to propositional forms the following points must be remembered.

1 The meaning of the original sentence must be faithfully preserved in the logical form too.

2 The proposition must express all its three parts in the proper order, viz. subject, copula and predicate.

3 The subject of the proposition can be found out by answering the question "Of what anything is being stated"

4 There must be a copula connecting subject and predicate.

5 When reducing a negative sentence to logical form. The sign of negation should go with the copula and with the predicate of the proposition.

6 Compound sentences must be split up in to simple sentences to construct propositions out of them.

7 The quantity of the propositions must be indicated clearly.

## 4.8 KEY WORDS

**Evocation:** Evocation is the act of calling or summoning a spirit, demon, god or other supernatural agent, in the Western mystery tradition. Comparable practices exist in many religions and magical traditions.

**Reduction:** Reduction in philosophy is the process by which one object, property, concept, theory, etc., is shown to be entirely dispensable in favor of another.

## 4.9 FURTHER READINGS AND REFERENCES

Copi, Irving M. and Cohen, Carl. *Introduction to Logic*. New Delhi: Prentice-hall of India Private Limited, 1997

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## 4.10 ANSWERS TO CHECK YOUR PROGRESS

**Check Your Progress I**

1 A proposition is the unit of thought and logic and carriers a definite truth-value. A proposition is expressed with the help of a sentence. While proposition is the unit of thought, sentence is the unit of grammar. The primary thing about the proposition is its logical form while for a sentence its primary thing is its grammatical form.

2 Aristotle has classified proposition into 4 kinds. They are as follows:

1 Universal affirmative (A Proposition)

2 Universal negative (E Proposition)

3 Particular affirmative (I Proposition)

4 Particular negative (O Proposition)

**BLOCK-2 INTRODUCTION**

Reasoning is the cognitive process of searching answers to problems, puzzles, etc. What is more astonishing is the fact that humans have rare ability to reason about reasoning itself. This two-fold function of reasoning is the kernel of philosophy. Different forms of reasoning occur in different fields. In philosophy, the study of reasoning typically focusses on what makes reasoning efficient or inefficient, appropriate or inappropriate, good or bad. Last but not the least, limits of reason is not ignored by philosophy. Philosophers attend to this programme by either examining the structure of reasoning within arguments, or by considering broader methods used to reach particular goals of reasoning. A philosopher's approach to reasoning is different from psychologist's approach. A psychologist is interested in knowing the way in which people reason, neural processes which are involved in reasoning, the influence of cultural factors etc. One approach to the study of reasoning is to identify various forms of reasoning that may be used to support or justify conclusions.

There are two main divisions in logic; deductive and inductive reasoning. Deductive logic characterises mathematical method.' Deductive logic, also known as formal logic, deals with distinction between correct reasoning and flawed or fallacious reasoning. The study of inductive reasoning is carried out within the field known as philosophy of science. The present block, consisting of 4 units, introduces these elements.

Unit1 deals with 'Meaning and Kinds of Reasoning.' In this unit you will become familiar with the role played by reasoning and inference in the development of logic. You will be exposed to objections to interpret logic as concerned with reasoning and inference and consequent shift in the understanding of the nature of logic.

Unit 2 highlights 'Deductive Reasoning.' It introduces you to one type of deductive argument known as immediate inference. A study of this topic demands that two key terms are explained; truth-conditions, which define, what are called, logical relations and second distribution of terms. Exhaustive account of traditional and modern analyses of distribution makes your study fruitful.

Unit 3 explains the meaning of 'Dilemma and Fallacies.' Through this unit you will become familiar with the limitations of dilemma. It also helps you to understand how a dilemma borders on abuse of reason. What fallacies reveal is startling. They help you to realise that while there are innumerable ways of reasoning wrongly, there is only one way of reasoning rightly. It is important to remember that in order to know what is right it is necessary to know what is wrong.

Unit 4 helps you to know something about the nature of science which is concealed in scholastic study of science carried out by practising scientists. This study is inward in the sense that it goes beyond physical laboratory. After you are through this unit you will realise that science has one more component, viz. intellectual laboratory a complement of the former.

## UNIT 1 MEANING AND KINDS OF REASONING

**Contents**



### 1.0 OBJECTIVES

In this unit you will become familiar with the role played by reasoning and inference in the development of logic. You will be exposed to objections to interpret logic as concerned with reasoning and inference and consequent change in the meaning of these words. The next section deals with intricate features of two forms of inference followed by the limits and attempts to justify criticisms made against these two forms. It may appear to be a repetition of what was said in the previous block. But it is not so. Discussion of these issues is an extension of earlier exposition. At the end of the unit you should be able to:

- distinguish classical logic from modern logic
- understand why modern logic deviated from the path of classical logic
- to make a subtle distinction between universalization and generalization
- make a close association between logic and mathematics

### 1.1 INTRODUCTION

It is profitable to contrast deductive inference with inductive inference within the framework of classical logic, before we undertake a detailed critical survey of induction within the gambit of contemporary philosophy. One of the characteristics of deductive logic is its formal character in virtue of its emphasis upon the structure and form of argument. It also functions as a ‘criterion of demarcation’, to borrow the phrase from Karl Popper, to distinguish deductive inference from inductive inference. Secondly, reasoning is one of the terms often used as synonymous with inference. Therefore it is desirable to consider various aspects of these two words also. Let us begin with the second aspect first.

### 1.2 MEANING OF REASONING AND INFERENCE

Reasoning consists, essentially, in the employment of intellect, in its ability to 'see' beyond, and 'within' as well, what is available to senses. Reasoning, therefore, can be regarded as an instrument which enables mankind to grasp 'unknown' with the help of 'known'. While reasoning can be regarded as an instrument, inference can be regarded as the process involved in extracting what is unknown from what is known. This is precisely the content of argument, the essence logic. And this is the way knowledge keeps growing.

## 1.3 OBJECTIONS AGAINST REASONING AND INFERENCE

Whatever is said about inference in this particular section applies more or less to reasoning. One argument which goes against inference is that it is beset with psychological overtones. What is afflicted with psychological overtones is essentially subjective. Logic, in virtue of its close association with knowledge, has nothing to do with anything that is subjective. Cohen and Nagel for this particular reason chose to use 'implication' instead of 'inference'. The difference can be understood easily when we look at the usage. Statements always 'imply' but do not 'infer'. I 'infer', but I do not 'imply'. Salmon fell in line with Cohen and Nagel when he said that the very possibility of inference depends upon reasoning. However, logicians like Copi, Carnap, Russell etc., chose to retain the word inference. But, all along, they only meant implication. Therefore keeping these restrictions in our mind let us freely use 'inference'.

Though the use of the word 'reason' is not much rewarding, the word 'reasonableness' has some weight. We often talk about reasonableness of conclusion. In this context reasonableness means 'grounds of acceptability'. Surely, in this restricted sense, reasonableness is objective just as inference is.

## 1.4 KINDS OF REASONING

If deductive logic is characterized by form, which also serves as a reliable 'criterion of demarcation' (1.0), then inductive logic must be characterized by something else (since reasoning, inference and logic are used as synonymous, any word can replace any other word). It is claimed that in inductive logic matter or content is primary as opposed to deductive logic. In order to understand the differences we must know a little about the nature of deductive and inductive inferences. These issues shall be addressed now.

When we deal with the form of deductive argument, we also deal with 'valid' and 'true', on the one hand, and 'invalid' and 'false', on the other. This particular distinction is very prominent. Only statements are true (or false) whereas only arguments are valid (or invalid). This distinction will take us to this table.

Table 1:

| | Statements | Arguments |
|---|---|---|
| 1) | True | Valid |

| | | |
|---|---|---|
| 2) | True | Invalid |
| 3) | False | Valid |
| 4) | False | Invalid |

This table helps us to understand the following distinction. a) A valid argument (1 and 3) may consist of completely true statements or completely false statements or both true and false statements. b) An invalid argument (2 and 4), similarly, may consist of statements in exactly the same manner mentioned above. Therefore it means that truth and validity may or may not coincide. Similarly, we have to distinguish between material truth and logical truth. Material truth is what is stated by matter of fact. Logical truth is the outcome of the structure of argument. We shall consider examples which correspond to four combinations (see table1). Let us call premises p1, p2, etc. and conclusion q.

Arg1:

p1: No foreigners are voters.
p2: All Europeans are foreigners.
q: ∴ No Europeans are voters.

Arg2:

p1: Some poets are literary figures.
p2: All play writers are literary figures.
q: ∴ some play writers are poets.

Arg3:

p1: All politicians are ministers.
p2: Medha Patkar is a politician.
q: ∴ Medha Patkar is a minister.

Arg4:

p1: 3 is the cube root of – 27.
p2: - 27 is the cube root of 729.
q: ∴ 3 is the cube root of 729.

(It is sufficient to accept that in the above mentioned argument all three propositions are false.)

These four arguments apply to serial numbers 1, 2, 3, and 4 respectively. First and third arguments have a definite structure in virtue of which they are held to be valid. While second and fourth arguments have a different structure which makes them invalid. When an argument is valid the premise or premises imply the conclusion. If there is no implication then the argument is invalid. Validity is governed by a certain rule which can again be represented in a tabular form. [Let us designate 'true' by 'T' (1) and 'false' by 'F' (0) as a matter convention].

Table 2:

| | p | q | |
|---|---|---|---|
| 1) | T(1) | T(1) | Valid |
| 2) | F(0) | F(0) | Valid |
| 3) | F(0) | T(1) | Valid |
| 4) | T(1) | F(0) | Invalid |

We can also say that the premises necessitate the conclusion. In this case, necessity is of a particular kind, i.e., it is logical necessity. Therefore, when there is implication, conclusion is necessarily true. Very often, deductive logic is identified with mathematical model. It is generally admitted that in both these disciplines information provided by conclusion is the same as the one provided by the premises. It means that both are characterized by material identity. Deductive argument, therefore, is an example for tautology. We say that an argument is tautologous when the combination of statements is true under all circumstances.

If, one can ask, the conclusion does not go beyond premises and no new information is acquired in the process, then why argue and what is the function of arguments? The answer is very simple. Knowledge is not the same as mere acquisition of information. Novelty is not a measure of knowledge. The legend is that Socrates extracted a geometrical theorem from a slave purported to be totally ignorant of mathematics. The moral is that knowledge is within, not in the sense in which brain or liver is within. Knowledge is the outcome of critical attitude. Knowledge is discovered, not invented and so goes the ancient Indian maxim: eliminate ignorance and become enlightened. If what is said is not clear, then consider this path. Deductive argument helps us to know what is contained in the premises, i.e., the meaning of the premises. It is an excursion into the analysis of the meaning of the premises. And the conclusion is an expression of the same. If so, it is easy to see how the denial of conclusion in such a case amounts to denying the meaning of the premises which were accepted earlier. What is called self-contradiction is exactly the same as the combination of denial of conclusion and acceptance of premises. Therefore we say that a valid deductive argument is characterized by logical necessity. Hence a deductive argument is tautologous. It means that it is always true.

At this stage, two terms will be introduced; *analytic* and *a priori*. Consider this example: 'all men with no hair on their heads are bald. We know that this statement is true in virtue of the meaning of the word 'bald'; not otherwise. Such a statement is called analytic. In such statements the predicate term (here 'bald') is contained in the subject term (here 'men with no hair on their heads'). Knowledge obtained from an analytic statement is necessarily a priori, i.e. knowledge prior to sense experience. In philosophical parlance all analytic statements are necessarily a priori. Deductive logic provides knowledge a priori, though the premises and conclusion considered separately are not analytic. However, deductive argument and analytic statement share a common characteristic; in both the cases denial leads to self-contradiction.
How do we say that deductive logic provides a priori knowledge? Consider an example.

Arg. 5: All saints are pious.
All philosophers are saints.
∴ All philosophers are pious.

Evidently, there is no need to examine saints and philosophers to know that the conclusion is true. Indeed, it is not even necessary that there should be saints who are pious as well as philosophers. This being the case, arg. 5 takes the following form without leading to distortion of meaning.

Arg. 5a: If all saints are pious and all philosophers are saints, then all philosophers are pious.
The argument is transformed into a statement which involves relation. All implicatory relations (the present relation is one such) are such that without the aid of sense experience, but with the

laws of formal logic alone, it is possible to derive the conclusion. Thus like an analytic statement, any valid deductive argument provides a priori knowledge and hence it is devoid of novelty. Deductive argument is properly characterized as logically necessary. It is improper to characterize deductive argument as absolutely certain. Being a priori is one thing and being absolutely certain is something different. At this point, it is not necessary to discuss in detail the differences between absolute necessity and logical necessity. It is sufficient to know that absolute certainty is not to be confused with logical certainty. While the former is not the same as a priori, the latter is. The difference is that absolute necessity is psychological and hence subjective whereas logical necessity is logical and hence objective.

When sense experience takes back seat, intellect or reason becomes the prime means of acquiring knowledge. Following the footsteps of Descartes, who is regarded as the father of rationalism, we can conclude that deductive logic is rational. So we have sketched three characteristics; logical necessity, a priori and rational. One character presupposes another because there is a thread which runs through these characteristics.

Deductive argument is characterized by qualitative difference in opposition to quantitative difference, i.e. differences between valid and invalid arguments are only in kind but not in degree. Let us make matters clear: a valid argument cannot become more valid in virtue of addition of premise or premises. On the other hand, if any one premise is taken out of a valid argument, then the argument does not become 'less valid'. On the contrary, it simply becomes invalid. So an argument is either valid or invalid. Validity is not a matter of degree. Therefore a valid argument is said to be satiated. This is what we mean when we say that the premises in a valid argument constitute necessary and sufficient conditions to accept the conclusion. An argument is invalid due to a 'missing link' in the class of premises.

We have learnt that validity is an important facet of deductive logic. Now it is time to understand the formal characters of deductive logic. Strawson lists three aspects of formal logic: generality, form and system. Generality is distinguishable, clearly, from matter. Generality means that individual is not the subject matter of logic. Formal logic concerns only with the relation between statements, but not objects. This is because it is futile to embark upon a study involving objects because such a study has only beginning but no end. Consider two examples,

Arg6:

p1: The author of Abhijnana Shakuntala was in the court of king Bhoja.
p2: Kalidasa is the author of Abhijnana Shakuntala.
q : $\therefore$ Kalidasa was in the court of king Bhoja.

Arg 6A:

p1: The author of Monadology was in the court of the queen of Prussia.
p2: Leibniz is the author of Monadology.
q: $\therefore$ Leibniz was in the court of the queen of Prussia.

It is easy to decide prima facie that the structure of these two arguments is identical. The difference consists in subject matter only and it is possible to construct countless arguments having an identical structure. Obviously, this is not a profitable exercise. The essence of formal logic consists in saying that p1 & p2 imply q or that q follows from or entails p1& p2. Only implication and entailment are relevant here. Strawson has made very clear this aspect. Implication or entailment is independent of subject matter. Therefore it is impossible to identify the subject matter in virtue of recognition of implication. This point can be further clarified with the help of variables. Let us represent Abhijnana Shakunthala or Monadology with x, Kalidasa or Leibniz with y and queen of Prussia or King Bhoja with z. Now the argument takes this form.

Arg6': p1: The author of x was in the court of z.
p2: y is the author of x.
q : ∴ y was in the court of z.

In this particular context, without knowing the contents of x,y,and z we can know that p1 and p2 together imply q. The same explanation holds good to any invalid or inconsistent argument.

Let us call such forms logical forms. A logical form has two components: variables and constants. x, y, z etc are variables. 'If …….then, or, and, not' and 'if and only if' are called logical constants. In the final analysis, the structure of an argument is determined by constants, but not variables. The dependence of the laws of an argument on constants can be illustrated in this way. In life science the classification of animals is an important topic. The anatomical features of birds and aquatic creatures differ and there is difference in the function of those organs. Just as birds have some organs in common, aquatic creatures have certain other organs in common. These common organs correspond to constants. Similarly, every class of argument has definite constants. Just as the structure of birds is different from the structure of aquatic creatures, the structure of one class of arguments is different from the structure of some other class of arguments. The laws which explain the function of the organs of birds are different from the laws which explain the function of the organs of aquatic creatures. Similarly, when the structure of an argument differs, the laws also differ.

Integration of rules is another characteristic of formal logic. The structures of argument and rules are mutually dependent. If it is possible to decide the structure of an argument and also different classes of arguments, then is possible to achieve what is called formalization or systematization.

Deductive argument is also regarded as demonstrative argument, because the premises offer conclusive evidences for the conclusion. Acceptance of premises leaves no room for any reasonable or meaningful doubt. On the contrary, induction stands for any non-demonstrative argument where the premises, irrespective of their number, do not and cannot offer conclusive evidences to the conclusion. The word 'induction' is the translation of what Aristotle called 'epagoge'. C.S. Peirce called them 'ampliative', because in this type of argument the conclusion always goes beyond the premises and the premises offer, at best, reasonable grounds to 'believe' such conclusion. Belief is not the same as proof, a distinction which was, more often than not, completely ignored by the protagonists of induction. This is one difference. Secondly, all characteristics of induction are opposed to deduction. Uncertainty and sense experience characterize any inductive argument. Let us consider the second character first. This type of argument begins with sense experience. The premises, therefore, can be called 'observation-statements which directly result from experience. However, the conclusion is not an observation statement because it overshoots the material provided by observation statements which is why the observation statements cannot justify the conclusion. No matter how many black crows I have seen, it cannot prove or justify that ‘all crows are black.’ In this example black crows which I have seen form the matter of observation statements. The statement ‘all crows are black’ not only includes observed crows, but also includes crows which have not been observed by me. It is the second component which is the root cause of endless debate on the nature of inductive inference.

At the outset, it is necessary to dispel a widespread and deep-rooted misconception. Inductive argument, it is held erroneously, always provides universal statement. On the contrary, what it provides is merely a statement which depends upon experience, but in itself is not an experiential statement. On some occasions, experience can vouch for the conclusion, but on some other occasions, it cannot. For example, considering the fact that, today I observed 5384 black crows, I may conclude that ‘tomorrow I will observe the same number of black crows’. This sort of conclusion is characterized by a sort of leap, leap from ‘observed to unobserved or unobservable’. This is called inductive leap which always results in generalization. Induction cannot even be conceived in the absence of generalization. Thus generalization is the hallmark of induction. However, a universal statement differs from generalization because it is possible to construct a universal statement within the limits of sense experience without involving generalization, for example, when I conclude after close scrutiny that every book in the library is a hardback edition, the conclusion is universal but it is not an instance of generalization because there is no leap from observed to unobserved or unobservable.

The example considered above is future-oriented and in principle, it is ‘verifiable’. However, inductive inference need not be so always. It can also be past-oriented which is surely, ‘unverifiable’. History, anthropology, Geology, etc. consist of arguments which are past-oriented. But the mechanism, involved in both the cases is exactly the same. Therefore the prime characteristic of induction is that the conclusion does not necessarily follow from the premises and that experience precedes inference which means that inductive inference is uncertain and *a posteriori*. Whatever knowledge we acquire ‘after experience’, or whatever depends upon experience is called *a posteriori* as opposed to a priori.

While logical certainty and a priori knowledge provided by deductive logic entitles it to be loosely called rational, uncertainty and a posteriori knowledge provided by inductive logic entitles it to be called empirical a character disputed by Popper. We will consider his arguments at a later stage. The uncertainty of inductive conclusion prompts us to introduce another basic term in philosophy, viz. ‘probability’. However, before we consider the probable nature of inductive conclusion some remarks on ‘content’ and ‘truth’ are needed.

Inductive inference is not formal in the sense that more than structure, the subject matter is relevant which is why the acceptability or relevance of the conclusion varies from one argument to another. Consider these examples:

Arg7:

> Over the years, the scientists compared finger prints of over a ‘million number’ of people and observed that none of them was identical with any other.
>
> ∴ No two finger prints in the world at any point of time are alike.

It is very easy to think that this particular conclusion is based on just one premise. In reality, it is based upon 'over a million number' of premises. A judgment on every pair of fingerprints is, indeed, a premise. Another important point to be noted is that this conclusion is not restricted by spatio-temporal factors. Compare the previous argument with this argument.

Arg8:

Thalidomide was administered to a large number of pregnant women as an antidote to morning sickness. In a significant number of those cases, infants developed physical deformity.

∴ This drug is likely to be harmful in future also.

The difference is that Arg7 does not include any exception whereas the Arg8 includes exceptions. Secondly, the former is taken to be beyond all reasonable doubts whereas the latter is not. Yet the second argument yields conclusion which is accepted in spite of contrary facts whereas the first argument yields conclusions which may be doubted. That it is not doubted is altogether different. It is possible that these two arguments enjoy credibility at different levels. What is important is that in none of these cases can we say that the conclusion is true because the premises do not imply the conclusion.

This analysis makes two points clear. Content alters the acceptability of argument and inductive argument is neither valid nor invalid. In other words, an inductive conclusion is neither true nor false. At best it is probable and at worst it is improbable.

Probability is a matter of degree. Assume that truth takes value '1' and falsity takes value '0'. Then the numerical value of probability varies from 0 to 1 without reaching either lower limit or upper limit. At this stage, it is enough to point out that the favourable premises raise the probability value. Therefore an inductive argument may consist of any number of premises, but what makes an argument more acceptable or less acceptable is the probability value that it takes. So we shall replace 'valid' and 'invalid' by 'good' and 'bad' and consequently, an inductive argument is either good or bad depending upon the level of its acceptability.

In deductive logic it is impossible to deny the conclusion, when the premises are accepted as true, without contradicting one's own self. How can an inductive argument escape from contradiction? The response is obvious. Conclusion includes more information than the premises. After accepting the premises if we deny the conclusion, we deny only that component of the conclusion which does not coincide with the premises. Therefore denial does not imply contradiction. The relation between premises and conclusion is very much akin to 'synthetic' as opposed to 'analytic' judgments where the meanings of subject and predicate are different, but otherwise related as in the case of the statement, 'The height of Mt. Everest is 29,000ft'. It is possible to ascertain the truth or falsity of such propositions, but it is not possible to know it a priori. So it was thought that all synthetic statements are necessarily *a posteriori* until Kant expressed his doubts on this issue. He tried to establish synthetic a priori propositions in order to counter Hume's attack on some metaphysical principles. Had he succeeded in doing so the development of inductive logic would have taken place in a very different direction.

## 1.5 ARGUMENTS AGAINST DEDUCTION AND INDUCTION

While deductive inference is exposed to less number of and less serious criticisms, induction is exposed not only to more serious criticisms, but also is attacked on more than one ground. While some of them find place in another unit, one particular criticism is considered here. Though this criticism was made by J.S. Mill with reference to one type of deductive argument known as syllogism, in general, any deductive argument is affected by this character. Mill contends that syllogism is guilty of repeating the premises in the conclusion without moving further. This criticism applies to inference within the limits of classical logic, where only true premises are considered or where the premises are taken to be true. When such premises and conclusion are conjoined, we get what is called compound statement and such statement is called tautology, because the same thing is said twice. The aim of logic is to achieve progress in knowledge. Deductive logic fails to achieve this particular aim. This objection can be effectively answered as has been pointed out early (1.4).

But the problem is more serious with induction. In the first place, induction is not regarded as logic at all since the truth of the conclusion does not follow necessarily from the truth of premises. Promptly, this objection was met by the defenders of induction by arguing that deductive standard ought not to be applied to inductive logic, lest the distinction itself becomes superfluous. As an alternative measure, some inductivists proposed what are called self-supporting inductive arguments. But any attempt to support one inductive argument with any inductive principle, if there is one, will, surely, lead to arguing in circle. This is so called because in this type of argument we are assuming what has to be proved which is a fallacy.

For quite some time it was believed that science follows a certain type of method which starts with observation of facts and ends up with generalization in the guise of law. This was the view of Bacon. Popper targeted induction precisely for this reason. While self-supporting inductive arguments involve arguing in a circle, any other attempt to justify induction results in infinite regress, i.e., if we use one principle to justify a law in science, then this principle stands in need of justification, and so on. This is what is known as infinite regress. These issues will engage our attention later (see unit 1 of block 3).

## 1.6 KINDS OF GENERALIZATION

While the type of deductive conclusion remains the same, the type of generalization differs. Broadly speaking, there are three types of generalization unrestricted generalization, restricted generalization and statistical generalization. Accordingly, induction also is of three types: unrestricted, restricted and statistical [instead of generalization, universal also can be used]. Generalization is said to be unrestricted when it does not include exception in any form. There are three types of restricted generalization; individual, spatial and temporal. Three illustrations are required to make this point clear.

1 Tendulkar will score a century in the next match.
2 All those who live in India are Hindus.
3 All those who lived before $20^{th}$ century were religious.

However, unrestricted generalization is free from any of these types of restrictions. The following statement illustrates this type.

4 All celestial bodies revolve in elliptic orbit.

Generally, restricted universal or generalization allows complete enumeration. But unrestricted generalization does not allow. Inductive logic in general and science in particular do not take enumeration seriously. Aristotle, indeed, regarded complete enumeration as one type of induction. It is important to note that complete enumeration does not generate generalization because there is no jump from 'observed' to 'unobserved'. Therefore it cannot even be regarded as inference. He was perhaps aware of this limitation. In some other place, he said that it is a kind of syllogism. Even then complete enumeration ceases to be induction. These problems forced Aristotle to propose another type which he called 'intuitive induction'. He defined it as '…a kind of induction which exhibits the universal as implicit in the clearly known particular'.

Analogy can be regarded as an example for intuitive induction. But the case of analogy is very different. Analogy excludes generalization of all types. Still, it is inductive, because with its help we pass from 'observed' to 'unobserved'. In this case, we notice certain similarities and over and above that a quality in one particular object but not in another. Then we infer that these objects (or persons) must be similar with respect to newly detected quality. This particular inference is, evidently, intuitive. Intuition is, essentially, subjective. But in this case the subjective nature of intuition does not pose any problem because what is inferred can be tested by anyone. Hence, analogy can be regarded as objective and also as inference.

Development in certain fields like statistics has given rise to a different type of generalization which may be called statistical generalization. Statistical generalization requires fair sample within which a study is undertaken yielding a certain ratio. This is, surely, an example for empirical approach. Observations made within this sample are extended to the parent class, i.e. the class of which the sample forms a part. It is quite likely that we may arrive at a certain ratio within fair sample whereas within the parent class we may arrive at some other ratio if certain other parameters influence the rest of the class. Another type of statistical generalization results when observations made in one sample become the ground to make observations in some other sample. In all such studies, it is frequency of occurrence of an event which matters. It is of utmost importance that in any statistical study fair sample should consist of elements selected by the same procedure.

In this context, a pertinent question arises. When does a sampling become fair? To be more precise, where can we draw the line demarcating fair sample from not so fair? To be sure, there is no such clear demarcation. Largely, it is a matter of convention which decides the fairness of a certain sample.

**Check Your Progress I**

**Note:** a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1) Analyse the relation between validity and formal character of deductive logic.

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2) Bring out the meanings of 'analytic and synthetic' and 'a priori and a posteriori'.

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3) Give examples (must be your own) for valid arguments consisting of only false statements and invalid arguments consisting of only true statements.

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4) Analyse the characteristics of induction.

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5) Comment upon the criticisms made against deductive and inductive inferences.

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6) Distinguish different types of generalization.

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## 1.7 LET US SUM UP

Inference or reasoning is of two types: inductive and deductive; deduction is formal and it is valid or invalid. Valid argument may consist of either true statements or false statements. Deductive inference is known a priori. Sense experience is irrelevant in deductive logic. Intellect is the key to deductive inference. Denial of conclusion leads to contradiction. Logical certainty, a priori nature and rational are the qualities of deduction.

Inductive inference is uncertain, *a posteriori* and empirical. Induction is regarded only by some as empirical. Inductive conclusion is the same as generalization. Generalization and universal are not same. In induction content determines acceptability whereas in deduction form determines validity. Probability is a matter of degree which is always a variable fraction. Deduction, it is said, is tautological whereas induction is neither an inference nor a method of science. Generalization is of three types: restricted, unrestricted and statistical.

## 1.8 KEY WORDS

**Axiom**: In traditional logic, an axiom or postulate is a proposition that is not proved or demonstrated but considered to be either self-evident, or subject to necessary decision. Therefore, its truth is taken for granted, and serves as a starting point for deducing and inferring other (theory dependent) truths.

## 1.9 FURTHER READINGS AND REFERENCES

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## 1.10 ANSWERS TO CHECK YOUR PROGRESS

**Check Your Progress I**

1) One of the characteristics of deductive logic is its formal character in virtue of its emphasis upon the structure and form of argument. Reasoning is also one of the formal characters of the deductive logic. Reasoning can be regarded as the process involved in extracting what is unknown from what is known. When we deal with the form of argument we also deal with 'valid' and 'true' on the one hand and 'invalid' and 'false' on the other. This particular distinction is very prominent. Only statements are true (or false) whereas only arguments are valid (or invalid).

2) we shall begin with an example: 'all men with no hair on their heads are bald ', We know that this statement is true in virtue of the meaning of the word 'bald'. Such a statement is called analytic. Knowledge obtained from an analytic statement is necessarily a priori, i.e. knowledge prior to sense experience. Deductive logic provides knowledge a priori, though the premises and conclusion considered separately are not analytic. However, deductive argument and analytic statement share a common characteristic. In both the cases, denial leads to self-contradiction. Any knowledge before experience is a priori and that knowledge which comes after experience is called a posteriori.

3) p1: All Indians are cricket lovers.
p2: Adolf Hitler is an Indian.
∴ Adolf Hitler is a cricket lover.
This argument is valid but the statements are not true.

p1: Some singers are musicians.
p2: All play writers are musicians.
q: ∴ some play writers are singers.
This argument is invalid but the statements may be true.

4) The word 'induction' is the translation of what Aristotle called 'epagoge". In this type of argument the conclusion always goes beyond the premises and the premises offer, at best, reasonable grounds to 'believe' such conclusion. Belief is not the same as proof, a distinction which was, more often than not, completely ignored by the protagonists of induction. This is one difference. Uncertainty and sense experience characterize any inductive argument. Let us consider the second character first. This type of argument begins with sense experience. The premises, therefore, can be called 'observation-statements which directly result from experience. However, the conclusion is not so because it overshoots the limits of observation statements which is why the observation-

statements cannot justify the conclusion. No matter how many black crows I have seen, it cannot prove that 'all crows are black.' The prime characteristic of induction is that the conclusion does not necessarily follow from the premises and that experience precedes inference, which means that inductive inference is uncertain and a posteriori. Whatever knowledge we acquire 'after experience', or whatever depends upon experience is called a posteriori as opposed to a priori.

5) Induction has attracted more number of criticisms than deduction. The criticism against deduction was made by J.S. Mill with reference to one type of deductive argument known as syllogism. However, in general any deductive argument is affected by this character. Mill contends that syllogism is guilty of repeating the premises in the conclusion. This criticism applies to inference within the limits of classical logic, where only true premises are considered or where the premises are taken to be true. When such premises and conclusion are conjoined we get what is called compound statement and such statement is called tautology, because the same thing is said twice. The aim of logic is to achieve progress on knowledge. Deductive logic fails to achieve this particular aim.

Induction, on the other hand is open to more serious criticisms. Induction is not regarded as logic at all since the truth of the conclusion does not follow necessarily from the truth of premises. Popper targeted induction for another reason. For quite some time it was believed that science follows a certain type of method which starts with observation of facts and ends up with generalization in the guise of law. This particular view came under attack by Popper.

6) There are three types of unrestricted generalization, restricted generalization and statistical generalization. 'Sachin will score a century in the next match' is a restricted generalization but 'all celestial bodies revolve in elliptic orbit' is the unrestricted generalization. Generalization is said to be unrestricted when it does not include exception in any form. Unrestricted generalization is free from any type of restrictions, whereas the other types are not. Restricted universal or generalization allows complete enumeration.

# UNIT 2 DEDUCTIVE REASONING

**Contents**



## 2.0 OBJECTIVES

This unit will help you to study:

- one type of deductive argument known as immediate inference
- the truth-conditions, which define the relations
- which terms are distributed or undistributed
- the subsequent units
- the limitations of immediate inference which have been briefly touched upon

## 2.1 INTRODUCTION

Since any reasoning involves argument, we shall begin with the type of relations between statements, which constitute an argument. A study of relation between propositions is also known as 'Eduction' in classical logic and eduction is one of the types of inference called immediate inference because we infer from one premise only.

## 2.2 DEDUCTIVE ARGUMENTS: TRUTH CONDITIONS OF RELATIONS

For the sake of convenience, let p and q denote premise and conclusion respectively. Any relation between p and q is defined by the truth-value, which they take. Let us first, define these relations. 'True' and 'False' are denoted respectively by 1 and 0.

1. p – 1, q – 1 } Relation
   p – 0, q – 1
   or
   p – 0, q – 0 } Independence
   p – 1, q – 0

2. p – 1, q – 1
   p – 0, q - 0 } Equivalence

3. p – 1; q – 0
   p – 0; q - 1 } Contradiction

4. p – 1; q – 1
   p – 0; q – Ind. } Superaltern

   Ind. stands for indeterminate.

5. p – 1; q – 0
   p – 0; q – Ind. } Contrary or contrariety

6. p – 1; q – Ind.
   p – 0; q – 1 } Subcontrary

7. p – 1; q – Ind.
   p – 0; q - 0 } Subaltern

No argument consists of independent statements. For example, a grouping of statements like 'Obama is the President of USA' and 'Grass is green' does not result in any type of argument. Therefore it does not interest us. Out of six relations, equivalence belongs to one category and relations from 3 – 7 belong to another category. Let us begin our study with the last category, which is known as 'opposition of relations'.

## 2.3 OPPOSITION OF RELATIONS

We shall establish various types of relation among four classes of categorical propositions. This study is restricted to categorical proposition.

1. **Contradiction**: This relation holds good for four pairs of propositions, which differ in quality and quantity.

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1. Premises | A | 'O' | E | I |

| Conclusions | 'O' | A | I | E |
|---|---|---|---|---|

Accordingly, if it is true that 'All rabbits are herbivorous' (RAH) -1, then it is false that 'some rabbits are not herbivorous (ROH) and if it is false that 'All rabbits are herbivorous' (RAH), then it is true that 'some rabbits are not herbivorous' (ROH). It is customary to represent the terms by the first letter of respective terms.

| | Statement | Symbol | Truth-Value | |
|---|---|---|---|---|
| 2. Premise | No statements are true. | SET | 1 | 0 |
| Conclusion | Some statements are true. | SIT | 0 | 1 |

| | Statement | Symbol | Truth-Value | |
|---|---|---|---|---|
| 3. Premise | Some crows are not black. | COB | 1 | 0 |
| Conclusion | All crows are black. | CAB | 0 | 1 |

| | Statement | Symbol | Truth-Value | |
|---|---|---|---|---|
| 4. Premise | Some lions are ferocious. | LIF | 1 | 0 |
| Conclusion | No lions are ferocious. | LEF | 0 | 1 |

2. **Contrary**: This relation holds good 'only' between universal propositions, which differ in quality.

| | Statement | Symbol | Truth-Value | |
|---|---|---|---|---|
| 5. Premise | All bats are mammals. | BAM | 1 | 0 |
| Conclusion | No bats are mammals. | BEM | 0 | Ind. |

| | Statement | Symbol | Truth-Value | |
|---|---|---|---|---|
| 6. Premise | No fish can fly. | FEF | 1 | 0 |
| Conclusion | All fish can fly. | FAF | 0 | Ind. |

3. **Superaltern:** When a particular conclusion is deduced from a universal proposition without affecting the quality, then superaltern relation holds good between the universal premise and particular conclusion. In this case the quality of proposition is irrelevant.

| | Statement | Symbol | Truth-Value | |
|---|---|---|---|---|
| 7. Premise | All metals are hard. | MAH | 1 | 0 |
| Conclusion | Some metals are hard. | MIH | 1 | Ind. |

| | Statement | Symbol | Truth-Value | |
|---|---|---|---|---|
| 8. Premise | No fruits are bitter. | FEB | 1 | 0 |
| Conclusion | Some fruits are not bitter. | FOB | 1 | Ind. |

4. **Subaltern:** When premise and the conclusion in superaltern are reversed we obtain subaltern. In other words, when we deduce universal conclusion from particular premise, the process results in subaltern.

| | Statement | Symbol | Truth-Value | |
|---|---|---|---|---|
| 9. Premise | Some planets are small. | PIS | 1 | 0 |
| Conclusion | All planets are small. | PAS | Ind. | 0 |

| | Statement | Symbol | Truth-Value | |
|---|---|---|---|---|
| 10. Premise | Some comets are not dense. | COD | 1 | 0 |
| Conclusions | No comets are dense. | CED | Ind. | 0 |

5. **Subcontrary:** This relation holds good between two particular propositions.

| | Statement | Symbol | Truth-Value | |
|---|---|---|---|---|
| 11. Premise | Some whales are heavy. | WIH | 1 | 0 |
| Conclusion | Some whales are not heavy. | WOH | Ind | 1 |

| | Statement | Symbol | Truth-Value | |
|---|---|---|---|---|
| 12. Premise | Some dogs do not attack. | DOA | 1 | 0 |
| Conclusion | Some dogs attack. | DIA | Ind | 1 |

It is important to notice asymmetry which results when we consider propositions differing in quantity. Keeping aside the relations, let us consider proposition pairs, which can be obtained by reversing the position of propositions found in the first pair. This reversal is coupled with the comparison of truth-values.

| | **Superaltern** | | **Subaltern** | |
|---|---|---|---|---|
| p: | 1 | 0 | 1 | 0 |
| q: | 1 | Ind. | Ind. | 0 |

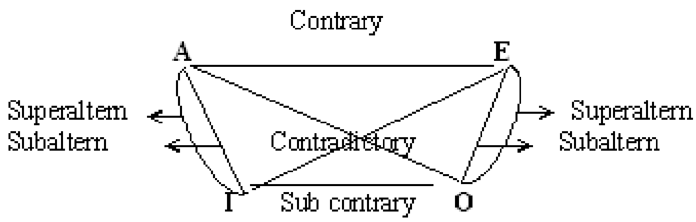
The truth-value of conclusion in superaltern differs from that of conclusion in subaltern. In other words, the relation, which connects universal premise with particular conclusion, is not the same as the relation which connects particular premise with universal conclusion. It is in this sense that between universal and particular there is asymmetry. In all these cases, the premises and the conclusion have the same subject and predicate. In fact, this is one of the preconditions to be satisfied to establish any opposition. The other condition may be stated as follows. Either quality or quantity, or both may be altered from premises to conclusion. Each change produces unique relation. In some cases, quantity of proposition determines the type of relation. This is how contrary and subcontrary can be accounted. Superaltern is understood in this way. A is the superaltern whereas I is its subaltern. Similar is explanation for E and O.

Traditional logic ignored asymmetry while identifying the relation. This is best explained with the help of a square:

1

F

2




1 r
c

**2**

S it
tl it
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] f
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A t.
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accurately. If subject has to be undistributed, then, it is necessary that the proposition should either include or exclude only 'some' members. When do we say or when are we allowed to use 'some'? Let the magnitude of S be x. Let S* (to be read s-star) denote the part of s, which is included in or excluded by a proposition. Now the formula, which represents undistribution of S can be represented as follows:

$$|x| > S^* \geq 1 \quad \ldots\ldots\ldots\ldots (1)$$

This is the way to read: "The modulus of x (|x|) is greater than S* greater than or equal to 1." It is highly rewarding to use set theory here. (1) indicates that S* is a proper subset of S. Therefore its magnitude must be smaller than that of x which is the magnitude of S. However, S* is not a null set. Because (1) shows that there is or exists at least one member in S*. Therefore in the case of undistribution the magnitude of S* varies between 1 and | x-1|. Now it is clear that in A and E, S (subject) is distributed while in I and O it is undistributed. Just to complete this aspect, let us state that all affirmative propositions undistribute P (predicate), whereas all negative propositions distribute P.

## 2.5 DIAGRAMMATIC PRESENTATION OF DISTRIBUTION

A better way of presenting distribution of terms was invented by Euler, an $18^{th}$ C. Swiss mathematician and John Venn a $19^{th}$ C. British mathematician. An understanding of the method followed by them presupposes some aspects of set theory.

Let S and P be non-null (non-empty) sets with elements as mentioned below (it is important that the status of set must invariably be mentioned, i.e., null or non-null). The following pairs shall be considered.

1. S = {a,b,c,d,e,f}, P = {g,h,i,j,k}

All letters within parentheses are elements of respective sets. In the first grouping there is no common element in these sets. Now, consider following groupings.

2. S = {a,b,c,d,e,f}, P = {a,b,c,d,e,f,g,h,i}
3. S = {a,b,c,d,e,f}, P = {b,c,d,g,h}
4. S = {a,b,c,d,e,f}, S* = {a,b,c}, P = {m,n,g,h}
5. S = {a,b,c,d,e}, P = {a,b,c,d,e}

Fifth group is unique in the sense that these two sets possess exactly the same elements. Therefore the magnitude of these sets also remains the same. Such sets are called identical sets. In 1908, Zermelo proposed what is called 'Axiomatic set theory'. One of the principal axioms in this theory is known as the Axiom of Extension or Extensionality. This Axiom helps us to understand the structure of identical sets. This theory was modified later by A Fraenkal and T. Skolem. Let us call this theory Zermelo – Fraenkel – Skolem set theory (ZFS theory). This theory states the above mentioned axiom as follows.

ZFS1: If a and b are non-null sets and if, for all x, $x \in a$ iff $x \in b$, then $a = b$

[Note '$\in$' is read 'element of' and 'iff' is read 'if and only if']
Symbolically, it is represented as follows:
$\{Sa\ \Lambda\ Sb\}\ \Lambda\ \{\forall x\ \ (x \in A <=> x \in b) => a = b$
This is the way to read:

| | |
|---|---|
| Sa | = a is a set |
| Λ | = and |
| ∀ | = for all values of |
| <=> | = if and only if |
| => | = if …then |

The summary of this formula is very simple. Whatever description applies to S (here a) also applies to P (here b). When distribution of terms is examined, the magnitude and elements of sets also are examined. Therefore it is wrong to assert that when S and P are identical sets, P is undistributed in A. Let us designate this type of proposition as A+ (read A cross). Consider these two propositions:

13. All bachelors are unmarried men. (BAU)
14. All spinsters are unmarried women. (SAU)

Knowledge of English is enough to accept that $B \equiv U$ and $S \equiv U$ ($\equiv$ reads identical).

First group corresponds to 'E'. When this group is compared with the remaining groups, it becomes clear that it differs from all other groups because in this group nothing is common to 'S' and 'P'. Consider 2nd 3rd and 4th groups. They correspond, respectively, to A, I and O. A brief description will suffice. In A proposition S is a proper subset of P, or P includes S, symbolized by

$S \subset P$ or $P \supset S$ ($\subset$ reads proper subset and $\supset$ reads includes)

The third group corresponds to 'I'. Here S and P intersect. So we have

$S \cap P = \{b,c,d\}$
($\cap$ reads 'intersect')

The fourth group requires some clarification. S* is incomplete, i.e., undistributed and P is completely excluded by S*. It means that 'O' distributes P.

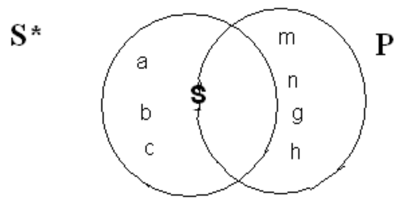
Let us return to E to which the first group corresponds.

$S = \{a,b,c,d,e,f\}$
$P = \{g,h,I,j,k\}$

N n
a s
c

△ △

N




S* = {a,b,c}
P = {m,n,g,h}

A brief explanation is required for SOP, which runs as follows:

1. S = {a,b,c,d,e,f}
2. S* = {a,b,c}; there is no information regarding d,e and f.
3. S* ⊆ S (S* ≤ S); S* is smaller than or equal to S
4. Let S – S* = S** (S** ≥ Φ )

'Φ' reads phi which stands for null set.

5. S** ⊆ P
6. S* || S**|| shows that elements of subsets S* and S** are different.

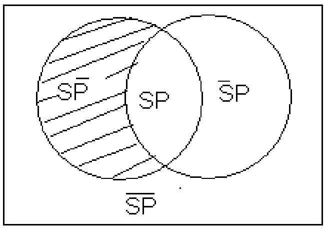
∴ S* || P

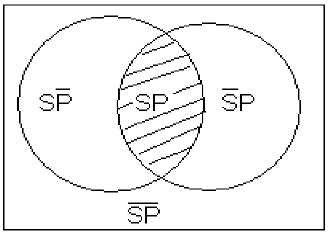
∴ Elements of S* and P are different.

John Venn (1834-1923) followed a very different method. Consider this proposition. All philosophers are simple = PAS. Since philosophers are humans, the universe of discourse is, obviously, 'humans'. Venn represents this with a rectangle. If philosophers are the elements of the Set P, then all humans other than philosophers constitute the complement of the set P. Complement of P is represented by $\overline{P}$ and the same explanation holds good for all classes. Now a new term is introduced, viz., '*product class*'. Any product class is an intersection of two or more than two sets (as far as logic is concerned, the number is restricted to three). PS is the product class of P and S. Such product classes may or may not be null sets. But $P\overline{P}$ and $S\overline{S}$ are null sets only which show that the product of a set and its complement is always a null set. When there are two terms, we get four product classes, which are as follows:

| | | |
|---|---|---|
| 1 | {PS} | Set of philosophers, who are simple. |
| 2 | {$P\overline{S}$} | Set of philosophers, who are not simple. |
| 3 | {$\overline{P}S$} | Set of humans other than philosophers, who are simple. |
| 4 | {$\overline{P}\,\overline{S}$} | Set of humans who are neither philosophers nor simple. |

It is pertinent to note that if there are three terms, then there are not six product classes, but eight product classes, i.e., if x is the number of terms, then $2^x$ is the number of product classes (i.e., 2×2×2=8). Now the time is ripe to introduce Venn's diagrams.


$S\bar{P}$
SP
$\bar{S}P$
$\overline{SP}$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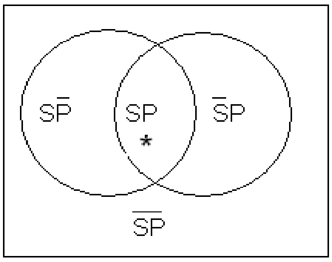
$\bar{S}P$
$\overline{SP}$


$S\bar{P}$
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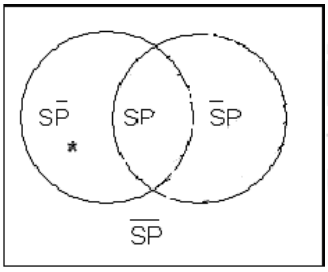
*
$\bar{S}P$
$\overline{SP}$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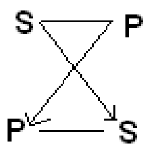
SP
$\bar{S}P$
$\overline{SP}$

$\bar{P}$ $\bar{P}$


S
P
P
S

only if it is distributed in the premise. However, a term, which is distributed in the premise, may or may not be distributed in the conclusion.

Now we shall convert A, E & I propositions. Later we will come to know that 'O' cannot be converted. In conversion, the conclusion is called converse.

| | Statement | Symbol | Truth-Value | |
|---|---|---|---|---|
| 15. | All stars are bright. | SAB | 1 | 0 |
| Stage:1 | bright stars<br>(Law 1: Transposition of terms) | B S | | |
| Stage: 2 | bright are stars<br>(Law 2: Constancy of quality, both premise and conclusion are affirmative) | | | |
| Stage: 3 | Some bright things are stars.<br>(Law 3: Terms undistributed in the premise (in this case 'bright' or 'bright things' remains undistributed in the conclusion. Also it is important to note that when an adjective becomes subject it is necessary to add appropriate countable noun like things.)) | BIS | 1 | 0 |

When A is converted it becomes I because the undistributed predicate (in this proposition bright) becomes undistributed subject after conversion. Since quantity changes from universal to particular this type of conversion is known as conversion per accidens. On the other hand, the conversion of E and I are simple because in these cases conversion does not require change of quantity.

| | Statement | Symbol | Truth-Value | |
|---|---|---|---|---|
| 16. | No criminals are saints. | C E S | 1 | 0 |
| Stage: 1 | saints criminals<br>(Law1: Transposition of terms) | | | |
| Stage:2 | saints are criminals.<br>(Law2: Constancy of quality, both premise and conclusion are affirmative) | | | |
| Stage: 3 | No saints are criminals.<br>(Law 3: Terms distributed in the conclusion are distributed in the premise as well). | S E C | 1 | 0 |

| | Statement | Symbol | Truth-Value | |
|---|---|---|---|---|
| 17. | Some books are useful. | B I U | 1 | 0 |
| Stage: 1 | useful books | U B | | |

| | | | | |
|---|---|---|---|---|
| | (Law 1: Transposition of terms) | | | |
| Stage: 2 | useful are books | | | |
| | (Law 2: Constancy of quality, both the premise and the conclusion are affirmative). | | | |
| Stage: 3 | Some useful things are books. | U I B | 1 | 0 |
| | (Law 3: Terms undistributed in the premise (in this case both S & P) are undistributed in the conclusion). | | | |

O does not have conversion, which needs some explanation. Consider this statement.

17. Some gods are not powerful.
    ∴ Some powerful beings are not gods.
    This conversion is invalid because the term 'gods' is distributed in the conclusion while it is undistributed in the premise. This type of conversion violates one of the laws of conversion, which stipulates that any term, which is undistributed in the premise, should remain undistributed in the conclusion. The term 'gods' can remain undistributed in the conclusion only if the conclusion is affirmative. If we obtain affirmative converse from a negative premise, then we violate another law of conversion, which stipulates that quality should remain constant. It only means that if 'O' is converted, then one or the other law of conversion is violated. Therefore 'O' has no conversion.

Conversion of 'O' and simple conversion of 'A' lead to a fallacy called fallacy of illicit conversion. Any fallacy in formal logic arises when any law or rule is violated. Suppose that the statement 'All Europeans are white' is converted as 'All white people are Europeans.' Then this conversion commits the fallacy of illicit conversion because in this example also the term white (or white people) is distributed in the conclusion while it is distributed in the premise. However, there is an exception to the restricted conversion of A, which we will examine later.

**Obversion**: This is one technique of preserving the meaning of a statement after effecting change of quality. The procedure is very simple; simultaneously change quality of the premise and replace the predicate by its complementary. We apply this law to the premises (A,E,I, and O) to obtain conclusions. The conclusion is called obversion.

| | | | | | |
|---|---|---|---|---|---|
| 18. | p: | All Players are experts. | P A E | 1 | 0 |
| | q: | ∴No players are non-experts. | P E $\bar{E}$ | 1 | 0 |
| 19 | p: | No musicians are novelists. | M E N | 1 | 0 |
| | q: | ∴All Musicians are non-novelists. | M A $\bar{N}$ | 1 | 0 |
| 20 | p: | Some scholars are women. | S I W | 1 | 0 |
| | q: | ∴Some scholars are not non-women. | S O $\bar{W}$ | 1 | 0 |
| 21 | p: | Some strangers are not helpful. | S O H | 1 | 0 |
| | q: | ∴Some stranger are non-helpful. | S I $\bar{H}$ | 1 | 0 |

D ... at
f ... r
p

A

$\bar{P}$

E

$\bar{S}$ ... $\bar{P}$

C ... $\bar{P}$ $\bar{S}$

D ... $\bar{S}$ $\bar{P}$

E ... $\bar{S}$

I ... o
c ... s
f

a

$\bar{P}$

b

$\bar{S}$ ... $\bar{P}$

c

$\bar{S}$ ... $\bar{P}$ $\bar{S}$

d

$\bar{S}$ $\bar{P}$

e ... e
o ... n
b ... d
c

L ... e.
F ... ll
r

A

Note: a) Use the space provided for your answer.
b) Check your answers with those provided at the end of the unit.

1. If it is true that 'no taxpayers are poor', then determine the truth-value of the following. Also, mention the relation with the given statement.
   a) All tax payers are poor.
   b) Some taxpayers are poor.
   c) Some taxpayers are not poor.
   d) All taxpayers are non-poor.
   e) No taxpayers are non-poor.
   f) Some tax payers are non-poor.
   g) Some tax payers are not non-poor.
   h) No poor people are taxpayers.
   i) All poor people are non-taxpayers.
   j) Some non-taxpayers are poor.

[N.B. It is not necessary that hyphen (-) should be used to obtain complement of the given term.]

2. Assume that the statement 'All players are sick' is false. Determine the truth-value of the following and state the relation with the given statement.
   a) Some players are sick.
   b) Some players are not sick.
   c) No players are sick.
   d) Some sick people are players.
   e) No players are nonsick.
   f) Some sick people are not nonplayers.
   g) All nonsick persons are nonplayers.
   h) No nonsick persons are players.
   i) Some nonplayers are nonsick.
   j) Some nonplayers are not nonsick.

3. Assume that the statement 'Some poets are philosophers' is true. Determine the truth-value of the following and state the relation with the given statement.
   a) Some poets are not philosophers.
   b) All poets are philosophers.
   c) No poets are philosophers.
   d) Some philosophers are poets.
   e) Some philosophers are not nonpoets.
   f) Some philosophers are not poets.
   g) All philosophers are poets.
   h) No philosophers are poets.
   i) All philosophers are nonpoets.
   j) Some philosophers are nonpoets.

4. Assume that the statement 'Some mangoes are sweet' is false. Determine the truth-value of the following and state the relation with the given statement.
   a) All mangoes are sweet.
   b) No mangoes are sweet.
   c) Some mangoes are not sweet.
   d) Some sweet things are mangoes.
   e) Some sweet things are not nonmangoes.
   f) No mangoes are nonsweet.
   g) Some nonmangoes are nonsweet.
   h) Some nonmangoes are not sweet.
   i) All mangoes are nonsweet.
   j) Some mangoes are nonsweet.

## 2.8 LET US SUM UP

One of the types of deductive inference is known as immediate inference. In this type we use one premise to derive conclusion. Traditional logic considers several varieties of deductive inference. They are also called relations. Each relation is uniquely defined by truth-conditions. Immediate inference provides an opportunity to understand how logic and mathematics have crossed. Apart from verbal description, diagrammatic representation also accurately describes these relations. A modern analysis helps us to understand the limitations and defects of traditional approach which results in the rejection of several relations as invalid which were, otherwise, accepted as valid.

## 2.9 KEY WORDS

**Modulus:** a modulus is a formal product of places of an algebraic number field. It is used to encode ramification data for abelian extensions of number field.

## 2.10 FURTHER READINGS AND REFERENCES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## 2.11 ANSWERS TO CHECK YOUR PROGRESS

(Note: '1' stands for 'true', '0'stands for false and '?'stands for doubtful.)

1.

a) '0'; contrary

b) ‘0’; contradictory
c) ‘1’; superaltern
d) ‘1’; obverse
e) ‘0’; contrary of obverse
g) ‘0’; contradiction of obversion
h) ‘1’; converse
i) ‘1’; full contraposition
j) ‘1’; converse of full contraposition

2.

a) ‘?’; superaltern
b) ‘1’; contradictory
c) ‘1’; contrary
d) ‘0’; converse
e) ‘0’; obverse
f) ‘0’; obverted converse
g) ‘0’; full contraposition
h) ‘1’; contrary of full contraposition
i) ‘0’; full inversion
j) ‘1’; subcontrary of full inversion

3.

a) ‘?’; subcontrary
b) ‘?’; subaltern
c) ‘0’; contradictory
d) ‘1’; converse
e) ‘1’; obverted converse
f) ‘?’; subcontrary of converse
g) ‘?’; subaltern of converse
h) ‘0’; contradiction of conversion
i) ‘0’; contradiction of obversion
j) ‘0’; subcontrary of obversion

4.

a) ‘0’; subaltern
b) ‘1’; contradiction
c) ‘1’; subcontrary
d) ‘0’;converse
e) ‘0’; obverted converse
f) ‘0’;obverse of subaltern
g) ‘0’;full inversion of subaltern
h) ‘0’;partial inversion of subaltern
i) ‘1’;obversion of contradiction
j) ‘1’; obversion of subcontrary

## UNIT 3 THE DILEMMA AND FALLACIES

**Contents**



### 3.0 OBJECTIVES

In this unit you are expected to understand

- dilemma which is not logically sound or acceptable .
- fallacies which would educate you on the pitfalls to be avoided in argument.
- why you are prone to err.
- your usual mistakes in argument.

### 3.1 INTRODUCTION

In the previous units, we learnt a good deal about categorical proposition, which is also known as unconditional proposition. In contrast to unconditional proposition, there is another class of proposition known as conditional proposition. In our study of symbolic logic, which will occupy us in the next units, we are required to make use of conditional proposition from a different perspective (within the framework of symbolic logic a conditional proposition is called compound proposition). Presently, we use conditional proposition to understand what the dilemma is.

There are two kinds of conditional propositions; hypothetical and disjunctive (Some authors like Cohen and Nagel use 'alternative' instead of 'disjunctive'. In the previous block also this distinction was made. However, here afterwards 'alternative' is not used. Instead, only 'disjunctive' is used.). Let us become familiar with the structure of these propositions.

1. If the economy of nation stabilizes, then inflation will dip.

A proposition, which has this particular structure, is called hypothetical. All hypothetical propositions consist of words 'if' and 'then'. Statement, which follows 'if' is called antecedent and statement, which follows 'then', is called consequent. In the given example antecedent and consequent are as follows.

| | | |
|---|---|---|
| economy of nation stabilizes. | – | antecedent |
| inflation will dip. | – | consequent |

When hypothetical proposition forms a part of an argument, it is stipulated that one of the two restrictions has to be followed; in second premise antecedent must be affirmed or consequent must be denied. Affirmation of antecedent leads to affirmation of consequent and denial of consequent leads to denial of antecedent. Denial of antecedent or affirmation of consequent anywhere else excepting in the conclusion is fallacious.

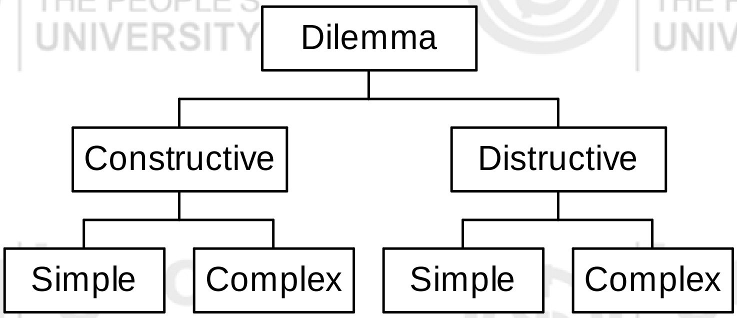
A disjunctive proposition does not have a fixed formation. It is identified with the help of words' either' and 'or'. It is not even necessary that both of them should be present. It is sufficient if 'or' alone is present. Consider these two examples.

2. A philosopher is either generous or a miser.
3. Education is useless or costly.

The only difference between 2nd and 3rd examples is that in the former, 'either' is explicitly stated, whereas in the latter, it is not. To avoid further confusion, let us stick to the former use. Generally, we deal with proposition, which has two components. A disjunctive proposition may have more than two components depending upon the type of 'or'. There are two types of 'or'; inclusive and exclusive. In 2nd example or is exclusive because it excludes third possibility, viz., being both generous and miser. Obviously, one who is generous cannot be a miser and vice versa. However, in the third example, 'or' is of inclusive type because education can be costly and useless in which case, the sentence is further extended as follows

3a Education is useless or costly <u>or both</u>.
Underlined segment of 3a refers to the extended part of 3. Since 'or' can be inclusive or exclusive, when it forms a part of argument, there should not be any confusion. Therefore, the rule stipulates that one of the components (they are called disjuncts) must be denied so that without any ambiguity the remaining disjunct can be affirmed.

One more aspect remains to be mentioned. Aristotle recognized one particular type of proposition as distinct. 'Socrates is mortal' is of this type. In this proposition, the subject does


Dilemma
Constructive
Distructive
Simple
Complex
Simple
Complex

consequents are affirmed in similar fashion. This description can be represented in this way:

$p_1$: $(p \Rightarrow q) \wedge (r \Rightarrow s)$
$p_2$: $p \vee r$
-------------------------------
q: $\therefore q \vee s$

2. In a simple constructive dilemma (SCD), both hypothetical propositions have common consequents, though antecedents differ. These antecedents are affirmed disjunctively in second premise and consequent is affirmed in the conclusion. Since there is only one consequent the conclusion is a simple proposition. The structure of this kind can be represented as follows

$p_1$: $(p \Rightarrow q) \wedge (r \Rightarrow q)$
$p_2$: $p \vee r$
-------------------------------
q: $\therefore q$

3. The structure of complex destructive dilemma (CDD) differs slightly from the first kind. The difference is that the disjunctive propositions in premise and conclusion negate disjunctively the components of respective propositions. However, the structure of the other premise remains the same. The form of CDD is mentioned below:

$p_1$: $(p \Rightarrow q) \wedge (r \Rightarrow s)$
$p_2$: $\neg q \vee \neg s$
-------------------------------
q: $\therefore \neg p \vee \neg r$

4. The structure of simple destructive dilemma (SDD) differs slightly from the second kind. In this type, also the conclusion is a simple proposition, but negative. The second premise has structure similar to that of $p_2$ of CDD. The form of SDD is as follows:

$p_1$: $(p \Rightarrow q) \wedge (p \Rightarrow r)$
$p_2$: $\neg q \vee \neg r$
-------------------------------
q: $\therefore \neg p$

Now, we can make a list of common features of different kinds of dilemma.

| | Dilemma | Common Features |
|---|---|---|
| 1. | Constructive | Different antecedents |
| 2. | Destructive | Different consequents |
| 3. | Complex | Disjunctive conclusion |

4. Simple Simple Conclusion

We will consider examples for four kinds, which can be used to illustrate these three methods.

i). Complex Constructive Dilemma (CCD):

p

$p_1$: **If** (any government wages war to acquire wealth), **then** (it becomes a

q r

rogue government) and **if** (it wages war to expand its territory), **then** (it

s

becomes colonial).

p r

$p_2$: (Any government wages war either to acquire wealth) **or** (to expand its

territory) q s

q: It (becomes a rogue government) **or** (colonial).

Formula: (p => q) Λ (r =>s)

p v r

---------------------

∴ q v s

ii). Simple Constructive Dilemma (SCD:

p q

$p_1$: **If** (taxes are reduced to garner votes), **then** (the government loses revenue)

r

and **if**(taxes are reduced in order to simplify taxation), **then** (the

q

government loses revenue).

p r

$p_2$: (Taxes are reduced either to garner votes) **or** (to simplify taxation)

q : ∴ (The government loses its revenue).

q

Formula: (p => q) Λ (r => q)

p v r

--- -------------------------

∴ q

iii). Complex Destructive Dilemma (CDD):

p q

$p_1$: **If** (the nation wages war), **then** (there will be no problem of

r

unemployment) and **if** (the nation does not revise her industrial policy),

s

**then** (it will lead to revolution).

⌐ q ⌐ s

$p_2$: The (problem of unemployment remains unsolved) **or** (there will not be any revolution).

⌐ p ⌐ r

q : ∴ (The nation does not wage war) **or** (the nation will revise her industrial policy).

Formula: $p_1$: (p => q) Λ (r => s)

$p_2$: ⌐ q v ⌐ s

q : ∴ ⌐ p v ⌐ r

iv). Simple Destructive Dilemma (SDD):

p q

$p_1$: **If** (you are in the habit of getting up early), **then** (you are a

r

theist) and (**If** (you are in the habit of getting up early), **then** (you are a labourer)

⌐ q ⌐ r

$p_2$: (you are not a theist) or(you are not a labourer)

⌐ p

q : ∴ (you are not in the habit of getting up early)

Formula: $p_1$: (p => q) Λ (p => r )

$p_2$: ⌐ q v ⌐ r

q : ∴ ⌐ p

## 3.4 AVOIDING DILEMMA

Use of dilemma is restricted to some situations. When neither unconditional affirmation of antecedent nor unconditional denial of consequents is possible, logician may use the dilemma. It indicates ignorance. When we face dilemma we only try to avoid, but not to negate. There are three different ways in which we can try to avoid dilemma. All these ways only reflect escapist tendency. Therefore, in logical sense, they do not carry much weight.

1. Escaping between the horns of dilemma: Two consequents mentioned may be incomplete. If it is possible to show that they are incomplete, we can avoid facing dilemma. This is what known as 'escaping between the horns of dilemma'. It should be noted that even when third consequent is suggested it does not mean that this new consequent is actually true. In other words, the new consequent also is hypothetical.

2. Taking the dilemma by horns: In this method of avoiding dilemma, attempts are made to contradict the hypothetical propositions, which are conjoined. A hypothetical proposition is contradicted when antecedent and negation of consequent are accepted. However, in this case this particular acceptance is missing. Instead, third component is offered to shield the antecedent after denying the consequent. Therefore contradiction is missing.

3. Rebuttal of dilemma appears to be its contradiction. But, in reality, it is not. In all these cases, the dilemma becomes a potent weapon to mislead the opponent in debate.

The first way of avoiding the dilemma, i.e., escaping between the horns of dilemma can be illustrated using 1 (CCD). It is possible to argue that, when the government wages war, the motive is neither to acquire wealth nor to expand its territory in which case, the government is neither rouge nor colonial. The motive may be to spread its official religion or personal vendetta or it may be to protect its interests. If the last one is the motive, then, it becomes difficult to find fault with such government. Any of the proposed alternatives to disjuncts may be false or all of them may be false. There is no way of confirming the same. The reader can select remaining examples to illustrate this method. Likewise, consider fourth argument to illustrate second method. I may concede that a person gets up early only because he wants to maintain health. So the purpose is not to worship God. Nor is he a labourer. Again, this is also an assumption.

Rebutting of dilemma requires a different type of example. Consider this one:

i).

$p_1$: If (teacher is a disciplinarian) [p], then (he is unpopular among students) [q] and if (he is not a disciplinarian) [$\neg p$], then (his bosses do not like him) [$\neg r$].

$p_2$: (Teacher is a disciplinarian) [p] or (he is not a disciplinarian) [$\neg p$].

q : $\therefore$ (Teacher is unpopular among students) or (his bosses do not like him).
q v $\neg r$

A witty teacher may respond in this way.

ii).

$p_1$ : if (teacher is not a disciplinarian) [$\neg p$], then (he is popular among students) [$\neg q$] and if (he is a disciplinarian) [p] then (his bosses will like him.) [r]

$p_2$ : (Teacher is not a disciplinarian) [p] or (he is a disciplinarian) [$\neg p$]

q : (Teacher is popular among students) [$\neg q$] or (his bosses will like him) [r]

Let us represent i & ii symbolically.

i). p1: $(p \Rightarrow q) \Lambda (\neg p \Rightarrow \neg r)$
$p_2$ $p \vee \neg p$
$\therefore$ ————————————
$q \vee \neg r$

ii) $(\neg p \Rightarrow \neg q) \Lambda (p \Rightarrow r)$
$\neg p \vee p$
————————————
$\therefore \neg q \vee r$

Now compare $(q \vee \neg r)$ and $(\neg q \vee r)$. Only a student of logic will realize that these two are not contradictories (you will learn about it in forthcoming units). Hence there is really no rebuttal.

Further, the dilemma, which an individual faces in day-to-day life, is very different; for example, moral dilemma. This has nothing to do with the kind of dilemma, which we have discussed so far.

**Check Your Progress I**

**Note:** a) Use the space provided for your answer.
b) Check your answers with those provided at the end of the unit.

1 How did the dilemma become popular? Why do we say that it is not an inference?
2 How many kinds of dilemma are there? What exactly can we achieve if we use any kind of dilemma?
3 Explain the structure of dilemma.

## 3.5 FALLACIES

Arguments are either valid or invalid. All valid arguments are good and invalid arguments are bad. A bad argument is also fallacious. Therefore, in the strict sense of the term, whatever causes an invalid argument also causes a fallacy. There are two ways in which an argument becomes fallacious; violation of any rule of inference results in fallacy. Secondly, In terms of truth-value of propositions, fallacy can also be caused by deducing false conclusion from true premise or premises. These two possibilities do not always overlap. Consider this example:

Some philosophers are not politicians.
Medha Patkar is not a philosopher.
∴ Medha Patkar is not a politician.

Although we have deduced a true conclusion from true premises, the argument is invalid and therefore, fallacious. A better way of interpreting fallacy is through integration of two ways of committing a fallacy. Accordingly, a fallacy is committed if we deduce a conclusion from given set of premises when it cannot be deduced. Copi has summed up this particular description in his analysis in his work *Introduction to Logic*, (9th Ed., Prentice Hall India, New Delhi, 1995, p.114). 'An argument whose premises do not support its conclusion, is one whose conclusion 'could' be false (or true) even if all its premises are true'. He goes a step further and defines 'fallacy as a type of argument that may seem to be correct, but that proves on examination, not to be so'. This particular type of definition is necessary, he argues, in order to distinguish persuasive arguments, which have only psychological force but not logical force.

Fallacies are many because there are many ways in which we may go wrong. While arguing we make mistakes sometimes consciously and sometimes inadvertently. These several fallacies are classified as follows (Edwards, Paul, ed. *Encyclopedia of Philosophy*. Vol 4. Macmillan and Free Press, 1972)); formal, informal, inductive and philosophical. First two types are deductive in nature and they are fallacious in the strict sense of the word. On the other hand, inductive fallacies can be regarded so only in a loose sense. This way of distinguishing is necessary because first two types of fallacies are committed when certain rules are violated and we are in a position to know clearly what those are. On the contrary, in case of inductive fallacy, there is no rule violated, or, at least, we are not in a position to decide with certainty whether or not any rule is violated. This confusion arises because in the first place, we are not sure whether there is anything like inductive rule just as there is something like deductive rule or rules. Yet, we are in a position to conclude, whether intuitively or not, that certain inductive arguments are acceptable (if not valid) and others are not acceptable (if not invalid). So, in a loose sense, we will say that inductive arguments, which are unacceptable, are fallacious. Philosophical fallacies are, in a sense, of a special variety, which arise due to very different reasons. The last type is not considered here.

## 3.6 FORMAL FALLACIES

Some formal fallacies deserve to be discussed under relevant units; for example, syllogistic fallacies. So it is apt to discuss them later. Fallacy of conversion was mentioned earlier. So it is also left out. Therefore we shall consider the remaining fallacies. In the previous section, we learnt about the structure of hypothetical proposition. It consists of antecedent and consequent. When it becomes a part of any argument or pseudo-argument, like dilemma, the application of rule becomes relevant. The rule stipulates that in second premise either antecedent must be affirmed or consequent must be denied. Consider the form of such arguments.

1) $p_1$: $p \Rightarrow q$
$p_2$: $p$
------ ---------
$q_1$: $\therefore q$

2) $p_1 : p \Rightarrow q$
$p_2 : \neg q$
----- ---------
$q2 : \therefore \neg p$

1) confirms that antecedent (p) is affirmed in second premise (p2) through which consequent (q1) is affirmed in conclusion. 2) shows that consequent (q) is denied in second premise (p2) through which antecedent (p) is denied in conclusion (q2). 1 is valid and the mood is known as *modus ponendo ponens* which means the mood in which 'something is affirmed (q) through

affirming something else (p). If ‘p’ is denied, instead of being affirmed, then the rule is violated and the argument commits the ‘fallacy of denying the antecedent’.
(2) is another valid mood. It is called *‘modus tollendo tollens’*. It means the mood in which something is denied (p) in the conclusion through something else being denied (q) in second premise. If ‘q’ is affirmed instead of being denied, then the argument commits the ‘fallacy of affirming the consequent’. The form of invalid arguments is as follows.

3) $p_1$: p => q
$p_2$: ⌐ p
-----------------
$q_1$: ∴ ⌐ q

4). $p_1$ : p => q
$p_2$ : q
-----------------
$q_1$ : ∴ p

The reader is advised to substitute statements for symbols.

Likewise, when disjunctive argument is involved the rules stipulate that one of the components must be denied in second premise through which the remaining component is affirmed in the conclusion. Consider this form:

p v q
⌐ q
--------------
∴ p

or

p v q
⌐ p
--------------
∴ q

There is no structural difference between these two examples. This is a valid mood, which is called modus *tollendo ponens* (mood in which one is denied and through which another is affirmed). When this rule is violated, the mood of the argument is *ponendo ponens* or *ponendo tollens*. Both are fallacious. Consider these forms.

p v q
p
--------------
∴ q

or

p v q
p
--------------
∴ ⌐q

It may be noted that *ponendo ponens* is valid only when hypothetical proposition is involved in the argument.

## 3.7 INFORMAL FALLACIES

We shall concentrate now on different class of fallacies. These are also called non-logical fallacies because there is no violation of any rule of inference as such. However, they are fallacious because in such arguments premises and conclusions are mutually irrelevant. Therefore they can also be called fallacies of irrelevance. Fallacy can also result due to ambiguity in language. There are in all sixteen such fallacies. A brief reference will be made to them.

1. *Petitio Principii* (Begging the question): In philosophical study, this fallacy is very common. It is committed when in our attempt to prove we assume what has to be proved. It means that something is proved on the basis of itself. We start from a position and end our argument by returning to the very same position. Hence *petitio principii* is also known as arguing in circle. An attempt to justify induction on the basis of some inductive principle is a classic example of this fallacy.
2. Accident: Fallacy of accident has two forms: direct fallacy and its converse. In both the cases, fallacy results due to inappropriate use of generalisation. So in order to distinguish former from the latter, the former can be qualified as direct accident. These fallacies are committed when the difference between normal and special circumstances is ignored. Since it can be ignored in two ways, we have two types of fallacies. When any norm, which applies to generalization is made applicable to any special case ignoring the difference between them, then fallacy of direct accident is committed. For example, murders are to be hanged, so all soldiers must be hanged.
3. Converse fallacy of accident: When the norm, which applies to a special case, is blindly extended to general circumstances, the converse fallacy is committed. An enlightening example is the dialogue between Socrates and Polemarchus. When Polemarchus argues that justice consists in repaying debt, Socrates promptly challenges him by demanding to know whether justice consists in returning arms, borrowed from my friend, to him when I know that he has passed from sober state to disturbed state. If the answer had been 'yes' then fallacy of direct accident would have been committed. If you argue in reverse order then converse fallacy is committed.
4. Argument *ad verecundiam*: This type of fallacy (and also next five types) is committed when we choose irrelevant premise. It is irrelevant because the premise really does not provide any support to the conclusion. This fallacy is committed when we try to get support from any person (usually famous and highly respected). Surely, from the point of view of logic, what a person says or does not say is irrelevant, more so when the person who is quoted is not an expert. This particular fallacy, in most of the cases, describes those who indulge in advertisement because in most of the advertisements the models who bat for the advertising companies know nothing about the products. Yet they speak with authority, which is endorsed by others.
5. Argument *ad Populum*: This fallacy is committed when a speech appeals to emotion and stirs up love or hatred. Generally, political speeches fall under this category. A classic example of this fallacy finds place in Shakespeare's Julius Caser, when Mark Antony instigates the crowd to take revenge on Caesar's killing. It should be noted that in such case the appeal is striking and hence it s noticeable easily.
6. Argument *ad Misericordiam*: This is an appeal to pity. From Plato's dialogues we understand that in ancient Greece, the criminals followed this method to escape punishment. It is doubtful whether this was followed by one who was not guilty.
7. Argument *ad Baculum:* Here, of course, there is no appeal but threat. Again, *baculum* is one method followed by those who are after power or who, supported by political authorities, try to enforce their ideology, whether religious or social. Threat may be to life or property or position. *Baculum* has all the features of totalitarian mindset and hence undemocratic.
8. Argument *ad Ignorantiam:* This is a commonplace fallacy committed in academic circles. *Udayana* a *vaisheshika,* argued that God exists because reason has failed to prove that God

does not exist. In other words, this fallacy is committed when I argue that my thesis is established when its antithesis could not be established by my opponent.

9. Argument *ad Hominem:* Arguments directed against the personality or character of the opponent, commit this fallacy. Again, this sort of fallacy is rare in academic circles, largely restricted to political circles. Generally, rivalry or animosity is behind committing this fallacy. Man is not always rational, no matter how deeply he is philosophized. Knowing fully well that it is unphilosophical to argue beside the point, we do so, generally, in a fit of rage.
10. *Ignoratio Elenchi:* There is a subtle difference between fallacies from 4 to 9 and this particular one. While in the case of former fallacies, the chosen premises are irrelevant, in the present case we get some other conclusion than the expected or intended one. In other words, instead of proving what is intended, we prove something different. It is not the case of missing the bus, but it is a case of the bus missing the route. It is a case of reasoning going 'astray'.
11. Complex question: Generally, complex question figures prominently in legal field. Complex question is an example of clever way of manipulation in order to checkmate the accused in particular or opponent in general. The question is framed in such a way that it admits only two answers and no matter which answer is chosen, the accused walks into the trap. The question is such that answers are hidden in it and hence it is impossible for anyone to construe any other answer to the question.

## 3.8 FALLACIES DUE TO AMBIGUITY

Ambiguity is of three types, use of ambiguous word, ambiguous structure of sentence and differing accent.

12. Equivocation is due to ambiguous words. 'Good' is one such ambiguous word. Consider this example: 'Rama is good'. 'Rama is a teacher'. Therefore, Rama must be a good teacher. It is one thing to be a good human being and something different to be a good teacher. This difference in the meaning of the word 'good' is obliterated here. Hence, fallacy of equivocation arises.
13. Amphiboly is due to the manner in which the words are combined and the hidden meaning which such combination suggests. The way in which Socrates understood what the oracle at Delphi said and the way in which others understood the same account for amphiboly. When the Oracle said that Socrates is the wisest man in Greece, Socrates took it to mean very differently.
14. Accent also can lead to fallacy. The premise emphasises one aspect while the conclusion emphasis another aspect. For example, when Jesus in his sermon, advises his disciples to 'love their neighbour' the advice could have been misconstrued by placing emphasis on the word 'your' while, in reality, Jesus emphasised the word 'neighbour'.
15. Composition: First fallacy consists in proceeding from parts to whole whereas the second consists in proceeding from whole to parts. Generally, these fallacies are committed when the attributes are under scrutiny. In the history of western philosophy we have famous

example of J.S. Mill who consciously committed the fallacy of composition. He said as follows:

Every man desires his own happiness

∴ All men desire the happiness of all.

16. Division: Composition and division are reciprocal fallacies. If the above-mentioned example is reversed with a little modification, then it becomes division.

No men desire the happiness of all.

∴ No man desires his own happiness.

It is obvious that fallacies of ambiguity are due to wrong interpretation or understanding, whereas logical fallacies are due to wrong reasoning. Interpretation and reasoning are different.

## 3.9 INDUCTIVE FALLACY

False cause: This fallacy consists in regarding an event as a cause of given effect, when, in reality, it is not the cause. But how are we to know that the supposed cause is not the cause at all? The only way is to wait for the occurrence of effect, which does not follow the supposed cause. But suppose that it did not happen. Then there is no way of deciding against the supposed link between cause and effect. For example, a historian may claim that the cause of India becoming independent is Second World War. There is no way in which the sequence of events can be repeated in future if this claim has to be tested. Hence, proof in the strict sense of geometrical proof is impossible in induction.

Secondly, there is no rule in inductive logic. Hence, there is no question of fallacy at all. With respect to inductive arguments, it can only be remarked that whatever opposes an acceptable inductive argument is fallacious. Mere common sense or experience is enough to suggest what is acceptable. Hence, without demanding logical proof, it is possible to decide what is acceptable and hence what is fallacious.

One advantage of knowing what fallacies are, whether in strict sense or in loose sense, is that if we know what is wrong, then we can correct mistakes or we may refrain from making them. And this is the way knowledge grows.

**Check Your Progress II**

**Note:** a) Use the space provided for your answer.
b) Check your answers with those provided at the end of the unit.

1 Distinguish between different kinds of fallacies.
2 Illustrate the fallacies of denying antecedents and affirming the consequent.
3 How many fallacies can occur in a politician's speech? Illustrate them.

## 3.10 LET US SUM UP

The dilemma is not an inference. Though it does not have any significance in logic, it is not possible to use it without any knowledge of logic. At least two propositions must be conditional (or compound). There are four kinds, complex constructive, and complex destructive on the one hand; simple constructive and simple destructive on the other. The dilemma is complex if the conclusion is conditional, otherwise simple. It is constructive when propositions affirm; otherwise destructive. We can only escape from or avoid dilemma. But it is not possible to disprove. Hence, it has use only in rhetoric. Knowledge does not owe anything to the dilemma.

Fallacies are of several types because mistakes are of several types. There are deductive fallacies, which are so in the strict sense. There are also fallacies in loose sense like inductive fallacies. Fallacies in formal and informal sense are due to mistakes we make in reasoning and interpretation. There are in all sixteen formal and informal fallacies and one inductive fallacy. There are two philosophical fallacies, which are different from logical. One advantage of knowing what fallacies are is that it helps in correcting the mistake or in avoiding the same.

## 3.11 KEY WORDS

When we are engaged in philosophical dispute, sometimes we attempt at solving the dispute without grasping correctly the nature of problem. This type of error is called philosophical fallacy.

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## 3.12 FURTHER READINGS AND REFERENCES

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## 3.13 ANSWERS TO CHECK YOUR PROGRESS

**Check Your Progress I**

1. The use of dilemma is an example of misuse or abuse of logic. Such a situation arises when a person, who is ignorant of logic, is confronted by an unscrupulous logician. It is most unlikely that the dilemma was seriously considered by any committed to logic. The dilemma, in the strict sense of the word validity, is neither valid not invalid. This is so because in this particular pattern there is no way of fixing the truth-value of the premises. The dilemma neither contributes to the growth of knowledge nor does it help in testing

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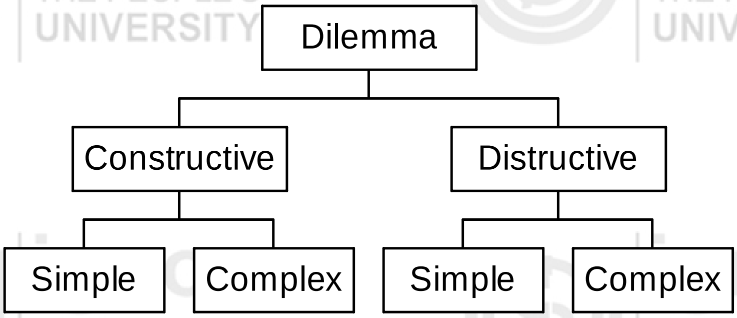


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4. The structure of simple destructive dilemma (SDD) differs slightly from the second kind. In this type, also the conclusion is a simple proposition, but negative. The second premise has structure similar to that of $p_2$ of CDD. The form of SDD is as follows:

$p_1$: (p => q) Λ (p => r)
$p_2$: ⌐ q v ⌐ r
------------------------------
q: ∴ ⌐ p

Now, we can make a list of common features of different kinds of dilemma.

| | Dilemma | Common Features |
|---|---|---|
| 1. | Constructive | Different antecedents |
| 2. | Destructive | Different consequents |
| 3. | Complex | Disjunctive conclusion |
| 4. | Simple | Simple Conclusion |

Use of any type of dilemma only helps us to evade a tricky situation. That is all what we can achieve with the help of dilemma.

3. The dilemma consists of three propositions of which two constitute premises and third one is the conclusion. The premises do not have any specific order. But the composition is fixed. One of the premises is a conjunction of two hypothetical propositions and the other one is disjunctive. The conclusion is either disjunctive or simple.

**Check Your Progress II**

1. Fallacies are many because there are many ways in which we may go wrong. While arguing we make mistakes sometimes consciously and sometimes inadvertently. These several fallacies can be classified as follows; formal, informal, inductive and philosophical. First two types are deductive in nature and they are fallacious in the strict sense of the word. On the other hand, inductive fallacies can be regarded so only in a loose sense.

2. Example for denying the antecedent and affirming the consequent:
   a) 1st premise: If(the government is weak), then (there will be anarchy)
      2nd premise: The government is not weak.
      Conclusion: Therefore there will be no anarchy.
   b) Example for affirming the consequent:
      1st premise: If (the government is weak), then (there will be anarchy)
      2nd premise: There will be anarchy.
      Conclusion: Therefore the government is weak.
3. Three types of fallacies are common in a politician's speech. They are as follows; baculum, hominem and populum. Hitler's speech instigated Nazis to kill Jews. Therefore

his speech is an example for baculum. When a politician makes public the immoral character of his opponent, his speech is an example for hominem. Mark Antony's speech which stirred up sympathy to Caesar in the minds of citizens leading to the murder of Brutus is an example for populum.

## UNIT 4 INDUCTION

**Contents**



## 4.0 OBJECTIVES

This unit brings to you a major philosophical issue which has run through ages; i.e., the problem of induction. A brief reference is made to all players who have contributed to the wealth of this issue. While Hume, Kant and Mill represent previous generation; Russell, Max Black and Popper represent the present generation. The link between induction and causation is shown in Mill's rejoinder to Hume. At the end, you should be in a position to appreciate Hume's arguments focusing on the perplexities of science and scientific method.

## 4.1 INTRODUCTION

Though Locke asserted that in the strict sense of the term, knowledge of natural laws is not possible, it was David Hume who discovered inherent problems in the structure of natural laws, which in other words, is known as empirical knowledge and method purported to be the method of science. It is desirable to note that these philosophers developed the criteria of empirical knowledge in opposition to the rationalistic conception of knowledge, which in course of time revolutionized our conception of science and scientific method.

But what is this so-called problem of empirical knowledge and what has this got to do with the problem of induction about which nothing is said at present? It is good to begin with Hume's fork. Hume says in his celebrated work, 'Enquiry concerning human understanding' that 'All objects of human reason or enquiry (knowledge) may naturally be divided into two kinds, '*relations of ideas and matters of fact*'. Hume conceded that the former kind is conclusively or demonstratively certain and hence can be denied only at the peril of self-contradiction. The so-called natural laws provide knowledge of matters of fact. Hence as things stand, this kind of knowledge is not certain, but can it somehow be made certain? This was Hume's problem. In other words, Hume gave a new twist to the problems which we face in our study of philosophy of science. His sole concern was to search a solution for the burning question - is scence a rational enterprise or just irrational, as irrational as a poet's imagination or hallucinations of a lunatic?

Though Hume never mentioned the problem of induction directly, his preoccupation with the problem of causation, which will be discussed later, has provided enough material to turn the heat on the supremacy of science through the problem of induction. In Hume's own words the problem of induction is "*Are we justified in reasoning from (repeated) instances of which we have experience to other instances (conclusions) of which we have no experience?*"

When Hume raised this question, little did he realize that he was opening the Pandora's Box. Since then a lot has been said on this problem. Subsequent discussion has not only resulted in diverse literature, but also it has given new dimensions, perhaps unforeseen by Hume himself, to the problem. So we consider Hume's problem '*classical'* or '*traditional'*, problem of induction to differentiate it from the later aspects of the problem.

Now, how does induction proceed or what is inductive method? As distinct from the deductive arguments, whose premises imply their conclusion, the premises of inductive arguments are merely supporting their conclusions; in other words, the conclusions of inductive arguments are probable, not necessary as in deductive argumentation. It is because inductive arguments proceed from particular instances to general propositions. In fact, deductive arguments get the general propositions through the inductive reasoning! For example, all humans are mortal, Ashok is human, therefore, Ashok is mortal, is a deductive argument; now, how is its first premise, 'all humans are mortal' derived? Certain kind of generalization is in the reasoning. The problem of induction consists in seeking out the possibility of deriving general principles from experiences. Now this procedure is questioned in the history from different corners.

## 4.2 KANT'S PROBLEM

In Unit 2 of block II, we discussed two sources of knowledge. The present discussion is an extension of previous discussion and both spring from Hume's fork. Kant, very soon, caught up with Hume's problem. In his own words, Hume woke him up from 'dogmatic slumber'. He adopted an ingenious method to get over the problem raised by Hume and hence save science from the ignominy of being irrational. The essence of the method is as follows. All analytic propositions without exception are a priori. Further, synthetic propositions are a posteriori. But the question is whether all synthetic propositions are a posteriori, or at least some are a priori. Kant answered the latter part of the question in affirmative. He was convinced that there are synthetic a priori propositions in mathematics. His only question was whether synthetic a priori propositions are possible in metaphysics too. He wrote 'The Critique of Pure Reason' with the intention of proving the possibility of synthetic a priori propositions in metaphysics. If he had succeeded in proving so, then he would have answered Hume effectively. One proposition which Kant wanted to show that it is synthetic a priori, is one connected to causation. History has recorded that Kant did not succeeded in his mission.

Kant's concern for synthetic a priori propositions, was, somehow, relegated to background in Germany. Instead, it gave birth to German idealism in Hegel. It was Mill who picked up the thread from where Kant had dropped. He, therefore, tried to answer Hume directly. Whatever development that took place in the problem of induction in the twentieth century has its origin in these three philosophers.

## 4.3 HUME'S ATTACK ON SCIENCE, VIS-A-VIS, INDUCTION

Broadly speaking, the problem of induction has divided philosophers into two camps; inductivists and non- inductivists or deductivists. Obviously, this is not exhaustive. In between we find sceptics and Hume in virtue of his mitigated scepticism heads this group. Even among inductivists who try to justify induction *there is no* unanimity. Among non- inductivists this problem does not arise because they simply refuse to accept induction as a form of logic.

The problem of induction deserves to be studied in all its complexity. Since whatever developed subsequent to Hume's arguments amounts to either partial acceptance of Hume's position or modifications of Hume's thesis, it is inevitable to begin with Hume's analysis of the problem. Hume took up seriously problems posed by ampliative induction which consists in arguing from '*observed to unobserved* or from '*observed to unobservable'*. For example, consider this argument.

Premise: For the past one month I have observed without fail that the bus which I take to go to the college arrives exactly at 8.30 A.M.
Conclusion: Therefore I conclude that on all days the bus arrives exactly at 8.30 A.M.

Hume's analysis reveals that there is no justification to argue from the past experience of the events of which we have positive information to the future events of which we have no experience whatsoever. The argument cited above is an example of induction which was criticized by Hume for its incompetence to stand the test of reason. But it is also a fact that given normal intelligence, every man in his senses argues and concludes on the basis of experience only. Experience may be his own or that of some other dependable person. Philosophy cannot afford to neglect such men. Hume says, therefore, that though there cannot be any logical justification for such a leap, it is possible to explain on practical grounds why men behave in such a manner. The repetition of events in a certain uniform manner induces men to react in the said manner. In Hume's own words, it is just a '*mental habit*' or mere '*animal instinct*' which is behind the scene. Custom is always devoid of reason and what Hume is exploring, are nothing but the principles of empirical sciences. To say that induction has its roots in custom is tantamount to saying that the methods adopted by the empirical sciences have their roots in custom. The upshot of Hume's thesis is that the irrationality of custom renders the whole of empirical science, which includes the much pampered experimental science, irrational, a pathetic conclusion indeed.

A little digression: When the problem of induction is treated as the problem of the methods of empirical sciences, there is no reason why the problem should not be considered the one belonging to the '*philosophy of science*'. Since a discussion of the problem of induction is also a discussion of the methods of empirical sciences, such a study is the study of the '*methodology*' of science or simply metascience, which also includes a discussion of the nature of laws and theories in science.

We shall put ourselves back on the track. Hume's argument has come in for many sharp criticisms. Obviously, for many, it is unpalatable to consider empirical sciences irrational; thanks to the sort of education they have received. Bertrand Russell, in no uncertain terms has exposed the situation in his work *History of Western Philosophy*. If '*the bankruptcy of eighteenth century reasonableness*' is not countered effectively, then, '*every attempt to arrive at general scientific laws from* singular observations is fallacious'. The question is how to redeem science from its impending grave. Thus we see that the problem of induction at once is transformed into a crucial problem of survival for science.

When Hume said that there can be no '*justification'* for induction, what did he mean by the word justification? Justification, in general, means satisfactory explanation. But Hume thought much more. He demanded that the explanation should be conclusive, or to put it in Hume's words, it should be 'intuitive or demonstrative'.

Barring Bertrand Russell, all other inductivists question the propriety of using reason the way Hume used. It is obvious that by adopting this function of reason as the criterion, Hume expected inductive inference to function as deductive inference. Possibly, it is only Russell who used the word reason in the sense in which Hume used. On the other hand, both Salmon and Urmson attribute Hume's failure to accept justification of induction to his reluctance or inability to distinguish inductive function from deductive function. Hume's attempt is somewhat analogous to an umpire judging cricket by applying the rules of hockey. It can be said, therefore, that Hume attempted an impossible task of deriving the synthetic proposition from an analytic proposition, or a posteriori from a priori.

There is another point in Hume's arguments which deserves to be mentioned. The problem of justification, which Hume raises, is purely a logical problem. But the answer provided by him to this question is psychological and hence the problem itself loses its logical character. Popper attempts at the elimination of this factor. We shall postpone our discussion of Popper's account to the next section.

Hume completes his analysis of the problem by pointing out that even probability of an inductive argument cannot be established beyond doubt. In other words, not only our '*expectation'* of the '*definite'* occurrence of an event, but also our expectation of the '*probable'* occurrence of an event has its roots in belief or custom. Therefore empirical knowledge can never free itself from custom. Therefore it is possible to conclude that the question raised by Hume is not a genuine question at all. Therefore various attempts have been made to justify induction on some other grounds. Once the grounds change, the formation of the problem also changes. The natural way to begin the investigation is by comparing inductive reasoning with deductive reasoning. How can we determine the validity of a deductive argument? Obviously, one way is through appeal to the deductive rules. If deductive rules help us to accept one argument and reject some other argument, then inductive rules, when precisely formulated, may help us to accept one inductive argument at the cost of some other argument. To consider the same analogy, if we can play hockey by following the rules of that game, we can as well play cricket by following the rules of that game and if there are no rules let us evolve them. Can we succeed in our mission?

## 4.4 IN DEFENSE OF INDUCTION

In his celebrated work, *The Problems of Philosophy*, Russell makes a distinction between two kinds of principles, which may be used to justify induction; empirical principle and non-empirical principle. According to him, an empirical justification of induction by appealing to facts is as good as an inductive justification of induction. Obviously, such a justification becomes circular. For example, in order to prove that all crows are black we cannot point to an observed sample and then proceed to unobserved because in such a case what is to be proved becomes the premise and the argument becomes circular. Accordingly, he formulated two principles of induction which are supposed to be non-empirical.

The greater the number of cases in which a thing of the sort A has been found associated with a thing of the sort B, the more probable it is (if no cases of failure of association are known) that A is always associated with B.
Under the same circumstances, a sufficient number of cases of the association of A with B will make it nearly certain that A is always associated with B.

Apparently, the word 'probable' or 'nearly certain' makes these principles non-empirical. But Russell's approach poses some problems. If Russell actually meant that a non-empirical principle should be adopted, then it necessarily means that induction has to be justified by invoking a non-inductive (non-empirical) principle. Hence, whatever charges are directed against Hume also can be directed against Russell. Secondly, Russell says that 'we want also to know that there is probability…… that things of the sort A are '*always'* B. Probability of A '*always'* becoming B does not seem to make sense. Suppose, for example, that there are only twenty swans in the world and I have seen fifteen swans, which are white. Now the chances or probability of any unobserved swan being white is 15/20 or 3/4. Russell's rule, now, is tantamount to saying that the probability of every unobserved swan being white is always 3/4. This is absurd. The very word probability should exclude 'always'. Russell's use of 'probable' indicates that it is the predicate of the event or object in question. But probability is only the relation between the premises and the conclusion. Thirdly, it is not necessary that all observed A should be B. Suppose that out of six observed swans 4 are white. Then the probability of any unobserved swan being white is 2/3. In other words, the probability of a general principle being true is 3/4, or 2/3, as the case may be.

However, according to a certain standpoint, the inclusion of the word probability saves inductive reasoning from one more crisis. It is said that to argue from 'all *observed* A are B' to the conclusion 'all A are B' amounts to formal invalidity because 'all *observed* A' do not equal 'all A', but less than that. In other words, the fallacy is similar to the one, which arises due to the violation of distribution rule. But if we say that 'all A are probably B', we are avoiding this fallacy.

But this argument is another instance, which illustrates the euphoria of deductive test. The very word formal invalidity, or validity, or reference to distribution rule shows either the supremacy of deductive test or the inability of the logicians to recognize that deductive reasoning is different from inductive reasoning. It seems, even Stebbing is unable (or may be reluctant) to recognize the difference.

The problem of induction was tackled by two groups of the sympathizers of induction on different lines. The analytic philosophers considered the problem itself a pseudo-problem. Paul Edwards and Urmson, for example, believed that the problem stands in need of reformulation. While Edwards was concerned with the analysis of the meaning of 'reason', Urmson thought that the traditional way of asking the question itself was wrong. According to Urmson, the transition from the question 'are any inductive arguments valid' to the question 'what good reasons can be given for rating arguments of a certain type higher than arguments of another type' is itself a real advance. But in his formulation there is a problem. When we raise the latter question, we presuppose that there is an argument, which is higher than the other, and then we search for the reasons, and hence the fallacy of complex question. Suppose that A and B are two arguments. How are we to know which is superior and which is inferior? We shall consider Urmson's analogy itself. How do we know the standards of a good apple? The answer is obvious; by considering taste, etc. But how do we know that a good apple should have a particular taste, should be free from wormholes, etc.? Obviously, mere reasoning about apple does not help. But experience is the guide. Either I should have tasted both the kinds of apple earlier, or I should have been told by someone. Urmson's formulation of the question includes neither of the alternatives.

Max Black is another philosopher whose account we shall consider now. He argues that support for inductive inference can be discovered within induction itself, but he contends that there will be no circularity contrary to the generally accepted view. To justify his stand, Max Black considers two orders of inference. He calls the inductive inference governed by an inductive rule a second order inference. Following his line of argument, we can call the inductive rule itself the first order inference. He affirms that as long as the first order inference differs from the second order inference there is no circularity. But how do we account for the difference? Again, the level of probability accounts for the difference. The inductive rule formulated by Max Black is as follows:

Rule: To argue from 'Most instances of A's examined in a wide variety of conditions have been B to (probably) the next A to be encountered will be B'.

It is sufficient if we note that in spite of Max Black's account of the difference between first order and second order inferences, Salmon and others contend that circularity is not avoided.

A common feature of this approach is to regard traditional formulation of the problem as defective because it arises due to conceptual or linguistic confusion and hence it is not a genuine problem. The concept here involved is 'justification' and consequently, reason. Paul Edwards is more critical about the word reason. In fact, his criticism of Russell's account is based on the meaning attributed by Russell to 'reason'. What has happened is that both Hume and Russell use the word 'reason' in a very different sense, different from common sense usage. This is the root of all problems. In common sense usage reason is mere ground for acceptance. For Hume and Russell reason is similar to logical proof.

In opposition to this approach, Salmon and Reichenbach developed the justification of induction on pragmatic lines. Salmon contends that the analytic approach to dissolve the problem is not a very satisfactory one. Here it is not the problem of justification, which matters, but the problem of inductive evidence matters. Whether or not the premises of an argument support the conclusion is based on whether the rule or rules governing the argument are correct or not. When there are several rules, either we should prove them or search some other means to choose the rules. The former is ruled out; hence only the latter remains. They adopt pragmatic standard to choose the best inductive rule from among many rules. This is a special form of justification, viz., vindication. Vindication is possible, when the rule achieves the purpose, which it is supposed to achieve.

After examining a few rules, both of them show that the rule of induction by enumeration is the most suited or the best possible rule. It is so because induction by enumeration (according to them) is functionally superior to any other rule. Now the shift from Hume is clear. The problem no longer is whether from the given true premises a true conclusion can be discovered in induction, but the problem is how to find out the best possible rule, and Salmon and others claimed to have achieved this and hence the solution to Hume's problem in its refined form. It may be noted that Russell also, in spite of his close affinity to Hume, tends towards pragmatic justification. The choice of a rule is similar to the choice of a theory. For example, we prefer Einstein's theory of gravitation to Newton's theory of gravitation because the scientific purposes are served better by the former. Such choice becomes a choice of community – the community of philosophers.

Since probability is at the root of all these discussions, something more remains to be said about it. Suppose that 'n' is the total number of events or objects in the class of 'A', and m is the number of observed events in the class. Probability, then, cannot be satisfactorily determined if n tends to infinity because in such a case the probability of the said event is nearly zero. J.M. Keynes, for example, asserts that a successful application of probability is possible only if we make certain presuppositions regarding the structure of the existent world. He considers two such principles, '*the Principle of Limited Independent Variety' and 'the Principle of Atomic Uniformity*'. According to the former, a limited number of objects with their finite number of qualities determine whatever variety we find in the given field. A very good example of this principle is the capacity of carbon to form as many as millions of compounds, and another is emphasis on the unified field theory. Even though we may not study all compounds of carbon, it is possible to assert a good deal about them with the help of our knowledge of carbon and other elements.The second principle only means that all macroscopic changes are the combinations of microscopic changes. Keynes admits that these principles are not logically necessary and hence he falls back on pragmatic considerations because they serve as the working principles in the field of science.

## 4.5 AGAINST INDUCTION

So far, we considered attempts to justify induction by its sympathizers. However, the most influential and at the same time devastating work on this problem is done by Popper. He takes a clear anti-inductivist stand. His attack on induction as a method of science begins with his analysis of Hume's problem. In one of his works, '*Objective Knowledge*', he attempts to replace all terms with psychological or subjective import in Hume's question regarding the justification of induction, by the so-called objective terms. For example, belief or reasoning is replaced by 'explanatory theory': instead of experience he uses the word basic or test statement. By effecting such changes, Popper believes that the possible danger of science being crippled by irrationalism can be averted.

Elimination of psychological terms is only the first step in this direction. In order to achieve total denial of induction, he denies induction by simple enumeration or repetition on the ground that it leads to infinite regress. Since we appeal to experience to justify induction, we have to recede from one event to another event without an end. We recede in the sense that we go back to the past instances one behind the other to justify the future. This regression never has an end. This is the second step. Thirdly, Popper rejects all possible rules of induction. In *Objective Knowledge* Popper considers the pragmatic justification of induction.

Upon which theory should we rely for practical action, from a rational point of view?
To replace this question Popper proposes an alternate question.
Which theory should we prefer for practical action, from a rational point of view?

Popper's contention is that in the second sense, i.e., for 'preference' no justification is needed. Hence we need not formulate any rule. If we consider an inductive rule in the first sense, it will be just false. Lastly, following Hume's line, Popper rejects the probability theory of induction and, instead, he uses corroboration as the principle. In simple language corroboration means support. Individual events can only support a theory but cannot prove the same.

Popper considers the problem of induction identical with the problem of demarcation. An important task of metascience is to draw a precise line of demarcation between science and metaphysics. Some analytic philosophers argue that all statements of empirical science, in opposition to metaphysical statements, should be capable of conclusive verification. Popper argues that neither induction nor conclusive verification provides a satisfactory line of demarcation. Hence, he evolved, following the lines of Whewell, another criterion of demarcation, viz., falsifiability. According to this criterion, a statement is empirical, if and only if it is capable of being falsified. He illustrates by considering the statement 'either it rains or it will not rain' and says it cannot be shown to be false and hence is not empirical.

## 4.6 FUNCTION OF FALSIFICATION

Popper claims that falsification is non-inductive in the sense that falsification is possible by adopting other method than induction, which he calls hypothetico-deductive method. This method consists in accepting tentatively a theory, i.e., a hypothesis, and then deducing logically conclusion from it. Repeated observations and experiments may confirm or falsify the theory. If confirmed, it is only temporary and falsification in future is not ruled out. If falsified, then it is final. When one hypothesis (conjecture) is falsified, we try another hypothesis. If this hypothesis should stand the test, it should succeed not only where the former has succeeded, but also should succeed, where the former has failed. However, Popper maintains that the emphasis should be on the attempt to falsify and not to verify. So it is imperative that we should invent the severest possible tests to falsify the hypothesis. Suppose that the hypothesis defeats all our genuine attempts to falsify. Only then it can be accepted. Then it is said to be corroborated. This is what Popper calls hypothetico-deductive method or simply deductivism, or trial and error method.

There is an important point to be noted. The traditional account of science states that the scientific approach to the problem begins with observation. Unless the events are observed the hypothesis cannot be formulated. But in Popper's analysis observation is not the source of hypothesis, but the means of testing and falsifying a hypothesis. In this manner Popper shifted the function of observation.

Corroboration is a technical term adopted in place of probability. The degree of corroboration is a function of the severity of the tests. At this stage, Popper links corroborability and improbability. If the content of a statement is minimal, then the statement is highly probable. Hence, falsification of such a statement is very difficult. For example, the statement, 'It will rain one day or the other at least in some corner of the earth', is certainly very highly probable. But the content is minimal. Hence not only falsification, but also corroborability is minimal. On the contrary, if the statement is dense, i.e., possesses maximum content, then it is highly improbable and in virtue of its very high improbability, it is capable of falsification and hence corroborable. Hence Popper adopts an extraordinary stand of preferring improbability, to probability. Significantly, the possibility of corroborability hinges on what is improbable becoming real. Falsification is regarded by Popper as the criterion of empiricism also.

Popper does not distinguish between a theory and a hypothesis. All theories are conjectural, and not all of them can survive. Popper endorses Darwinian theory. The fittest theory is that which has, so far, withstood the tests. Only that theory survives and other theories are rejected resulting in their extinction and hence he considers his method evolutionary.

We shall conclude by referring to Kant's problem again, viz., how are synthetic a priori judgments possible in metaphysics? Kant said that these judgments are imposed on nature by the knowing mind. But without stopping there he further argues that precisely due to their origin in the functions of our mental faculties that these judgments are absolutely certain. So it is obvious that the very nature of Kant's a priori is functional. Popper's view is that the synthetic a priori judgments are '*genetically a priori but not valid a priori*'.

**Check Your Progress**

**Note:** a) Use the space provided for your answer.
b) Check your answers with those provided at the end of the unit.

Bring out Hume's observations on induction.

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Examine various attempts to justify induction.

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Examine Poppers objections to regard induction as a method of Science.

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## 4.7 LET US SUM UP

Hume raises objections to one form of induction known as ampliative induction. The objections to generalization are formulated on one ground; no proof in strict mathematical or deductive sense is possible when we deal with induction. Inductive inference can only be vindicated because any attempt to justify the same runs into infinite regress or becomes circular. Generalization can only be disproved or falsified though it cannot be proved. Salmon, Max Black and Urmson defend induction whereas Russell and Popper reject induction. Popper's theory is known as anti-inductivism or non-inductivism. Popper replaces verifiability by falsifiability.

## 4.8 KEY WORDS

**Hypothesis:** A hypothesis consists either of a suggested explanation for an observable phenomenon or of a reasoned proposal predicting a possible causal correlation among multiple phenomena. The term derives from the Greek, *hypotithenai* meaning "to put under" or "to suppose."

**Probability** is a technical term used extensively in all sciences which use quantitative analysis. It, generally, is used to deal with projection which is future-oriented or past-oriented. It is always expressed in proper fraction where the denominator points to the total number of possibilities and numerator points to the issue at stake.

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## 4.10 ANSWERS TO CHECK YOUR PRGRESSS

Hume argues that there is no justification to argue from the past experience of the events of which we have positive information to the future events of which we have no experience whatsoever. But it is a fact that given the normal intelligence, every man in his senses argues and concludes on the basis of his experience. Philosophy cannot afford to neglect such men. Hume says, therefore, that though there cannot be any justification for such a leap, it is possible to explain why men behave in such manner. The repetition of events in a certain uniform manner induces men to react in the said manner. In Hume's own words, it is just a '*mental habit*' or mere '*animal instinct*' which is behind the scene. Custom is always devoid of reason and what Hume is exploring, are nothing but the principles of empirical sciences. To say that induction has its roots in custom is tantamount to saying that the methods adopted by the empirical sciences have their roots in custom. The upshot of Hume's thesis is that the irrationality of custom renders the whole of empirical science, which includes the much pampered experimental science, irrational, a pathetic conclusion indeed.

The problem of induction was tackled by two groups of the sympathizers of induction on different lines. The analytic philosophers considered the problem itself a pseudo problem. Paul Edwards and Urmson, for example, believed that the problem stands in need of reformulation. While Edwards was concerned with the analysis of the meaning of 'reason', Urmson thought that the traditional way of asking the question itself was wrong. According to Urmson, the transitions from the question 'are any inductive arguments valid' to the question 'what good reasons can be given for rating arguments of a certain type higher than arguments of another type' is itself a real advance. But in his formulation there is a problem. When we raise the latter question, we presuppose that there is an argument, which is higher than the other, and then we search for the reasons, and hence the fallacy of complex question. Suppose that A and B are two arguments. How are we to know which is superior and which is inferior? We shall consider Urmson's analogy itself. How do we know the standards of a good apple? The answer is obvious; by considering taste, etc. But how do we know that a good apple should have a particular taste, should be free from wormholes, etc.? Obviously, mere reasoning about apple does not help. But experience is the guide. Either I should have tasted both the kinds of apple earlier, or I should have been told by someone. Urmson's formulation of the question includes neither of the alternatives.

Max Black is another philosopher whose account we shall consider. He argues that support for inductive inference can be discovered within induction itself, but he contends that there will be no circularity contrary to the generally accepted view. To justify his stand, Max Black considers two orders of inference. He calls the inductive inference governed by an inductive rule a second order inference. Following his line of argument, we can call the inductive rule itself the first order inference. He affirms that as long as the first order inference differs from the second order inference there will be no circularity. But how do we account for the difference? Again, the level of probability accounts for the difference. The inductive rule formulated by Max Black is as follows:

Rule: To argue from 'Most instances of A's examined in a wide variety of conditions have been B to (probably) the next A to be encountered will be B'.

It is sufficient if we note that in spite of Max Black's account of the difference between first order and second order inferences, Salmon and others contend that circularity is not avoided.

A common feature of this approach is to regard traditional formulation of the problem as defective because it arises due to conceptual or linguistic confusion and hence it is not a genuine problem. The concept here involved is 'justification' and consequently, reason. Paul Edwards is more critical about the word reason. In fact, his criticism of Russell's account is based on the meaning attributed by Russell to 'reason'. What has happened is that both Hume and Russell use the word 'reason' in a very different sense, different from common sense usage. This is the root of all problems.

In opposition to this approach, Salmon and Reichenbach developed the justification of induction on pragmatic lines. Salmon contends that the analytic approach to dissolve the problem is not a very satisfactory one. Here it is not the problem of justification, which matters, but the problem of inductive evidence. Whether or not the premises of an argument support the conclusion is based on whether the rule or rules governing the argument are correct or not. When there are several rules, either we should prove them or search some other means to choose the rules. The former is ruled out; hence only the latter remains. They adopt pragmatic standard to choose the best inductive rule from among many rules. This is a special form of justification, viz., vindication. Vindication is possible, when the rule achieves the purpose, which it is supposed to achieve.

After examining a few rules, both of them show that the rule of induction by enumeration is the most suited or the best possible rule. It is so because induction by enumeration (according to them) is functionally superior to any other rule. Now the shift from Hume is clear. The problem no longer is whether from the given true premises a true conclusion can be discovered in induction, but the problem is how to find out the best possible rule, and Salmon and others claimed to have achieved this and hence the solution to Hume's problem in its refined form. It may be noted that Russell also in spite of his close affinity to Hume, tends towards pragmatic justification. The choice of a rule is similar to the choice of a theory. For example, we prefer Einstein's theory of gravitation to Newton's theory of gravitation because the scientific purposes are served better by the former. Such choice becomes a choice of community – the community of philosophers.

The most influential and at the same time devastating work on induction is done by Popper. He takes a clear anti-inductivist stand. His attack on induction as a method of science begins with his analysis of Hume's problem. In one of his works, '*Objective Knowledge*', he attempts to replace all terms with psychological or subjective import in Hume's question regarding the justification of induction by the so-called objective terms. For example, belief or reasoning is replaced by 'explanatory theory': instead of experience he uses the word basic or test statement. By effecting such changes, Popper believes that the possible danger of science being crippled by irrationalism can be averted.

**BLOCK-3- INTRODUCTION**

Logic deals with statements that vary extensively in the precision with which they may be made. If someone says, "That is a good book," it is a statement. It is far less precise, however, than a statement such as "New Delhi is the capital of India." A good book could be good because it is well printed and bound. It could be good because it is written in good style. It could tell a good story, etc. The statements that logic handles with the greatest certainty are those that obey the law of the excluded middle, i.e., those which are unambiguously true or false, not somewhere in between. By a "statement" in logic one means an assertion which is true or false. One may not know whether the statement is true or false, but it must be one or the other. The present block, consisting of 4 units, deals with induction and various forms of statements in symbolic logic.

Unit 1 is on "History and Utility of Symbolic Logic." In this unit, an attempt is made to present a history of symbolic logic. Since it is improper to make a beginning from middle, a brief mention of the history of ancient logic is also undertaken. You will be quick enough to notice that the moment you enter symbolic logic, you are confronted with mathematics as well.

Unit 2 highlights "Compound Statements and their Truth-Values." In this unit an attempt is being made to project the structure of and variety in proposition in a new perspective. Secondly, two shades of meaning of compound statement will be distinguished in order to accommodate one type of statements, which looks simple. A clear definition of the truth-function is attempted by considering two parameters simultaneously.

Unit 3 explains the meaning of "Syllogism," which is a mediate inference. In this unit an attempt is made to introduce the salient features of syllogism, which form an important part of Classical or Aristotelian Syllogism. Apart from providing traditional account of syllogism, sincere efforts are put to integrate traditional analysis with modern analysis. In doing so, some vital differences between these analyses are brought to the fore.

Unit 4 discusses "Truth-Functional Forms." The aim of this unit is to introduce you to the concept of equivalence in two ways: truth-table method, stroke and dagger function, and contradiction through truth-table means. Though what you learn in this unit is much limited in terms of content, it forms the foundation of future learning. Hence, this unit should prepare you to grasp the essence of the next block.

The 3 units given above will give you basic understanding of symbolic logic that can be used to translate verbal statements in ambiguous form into an exact mathematical form. Here we can discern three stages in the problem formulation: the verbal stage, the logical stage and the mathematical stage. As we come to know shortly symbolic logic begins with propositional calculus. Compound propositions are characterized by both variables and constants. Contradiction and equivalence are two important logical relations. While conjunction and bicondition do not have equivalent forms other compound propositions have equivalent forms.

# UNIT 1 HISTORY AND UTILITY OF SYMBOLIC LOGIC

**Contents**



## 1.0 OBJECTIVES

In this unit, an attempt is made to present a history of symbolic logic. You will be quick enough:

to notice that the moment you enter symbolic logic, you are confronted with mathematics as well.
to learn that development of logic and mathematics are inseparably related.
to know that logic and mathematics are two components of one enterprise.
to be familiar with conceptual developments with a brief description of what they are.
to set your priorities right, to identify the elements of logic in mathematical discussions.

## 1.1 INTRODUCTION

History of logic can safely be divided into three phases; ancient logic, medieval logic and modern logic. It is necessary to bear in mind that one is not just replacement for the other and that elements of later phase can be discerned in the earlier phase. Therefore development is significantly in terms of correction and improvements, but not total rejection. Therefore it is absolutely necessary to admit that the limitations of ancient and medieval systems of logic paved way for the rise of symbolic logic and its value in addition to pioneering work by some mathematicians.

## 1.2 EARLIEST CONTRIBUTION TO LOGIC

The greatest contribution of Aristotle to logic, undoubtedly, is his theory of syllogism in which the theory of classes and class relation is implicit. Another significant contribution of Aristotle is his notion of variables. Classes themselves are variables in the sense that in any proposition subject and predicate terms are not only variables but also they are the symbols of classes.

Finally, the class relation, which is explicit in his four-fold analysis of categorical proposition, is understood as inclusion or exclusion - total or partial.

Theophrastus, a student of Aristotle, developed a theory of pure hypothetical syllogism. A hypothetical syllogism is said to be pure if all the three propositions are hypothetical propositions. Theophrastus showed that pure hypothetical inference (an inference which consists of only hypothetical propositions) could be constructed which corresponds to inference consisting of only categorical propositions (which Aristotle called syllogism).

A school of thought flourished during Socrates' period known as Megarians. The first generation of Megarians flourished in the 5th century B.C. onwards. In the 4th century B.C. one Megarian by name Eubulides of Miletus introduced now famous paradox – the paradox of liar. The last Greek logician, (who is also 'lost' because none of his writings is extant), who is worthy of consideration is Chrysippus of whom it is said that even gods would have used the logic of Chrysippus if they had to use logic.

Peter Abelard, who lived in the 11th Century, is generally regarded as the first important logician of medieval age followed by William of Sherwood and Peter of Spain in the 13th Century. They continued the work of Aristotle on categorical proposition and syllogism and other related topics. In reality, no vacuum was created in medieval age and hence there was continuity from Aristotelian logic to modern logic though no original contribution came from any logician. The most notable contribution to logic in this period consists in the developments, which took place in several important fields like analysis of syntax and semantics of natural language, theories of reference and application, philosophy of language, etc., the relevance of which was, perhaps realized only very recently. These are precisely some of the topics of modern logic. William of Sherwood and Peter of Spain were the first to make the distinction between descriptive and non-descriptive functions of language. They reserved the word 'term' only for descriptive function. Accordingly, only subject and predicate qualify for descriptive function and hence in categorical proposition we can find only two terms. These were called categorematic whereas other components of a sentence like 'all, some, and no', etc. were called syncategorematic. The former are terms whereas the latter are only words. Hence, terms were regarded as special words. It is in this context that the medieval logicians made semantic distinction of language levels. Categorematic term was divided into two classes, terms of first intension and terms of second intension. First class stands for things whereas the second stands not for a thing but for a language sign. In a limited sense, and at elementary level, it can be said that subject represents first class and predicate represents second class. Another field covered by medieval logicians was that of quantification which is of great importance in modern logic. again, this is another important topic of modern logic.

## 1.3 LIMITATIONS OF ARISTOTELIAN LOGIC

The very fact that Aristotle constructed an extraordinarily sound system of logic became its nemesis. Just as Newtonian Physics was held as infallible for a little more than two hundred years, Aristotle was held on similar lines for nearly two thousand years. However, neither of them anticipated this treatment to their systems. While this is one reason for the delayed beginning of modern logic, second and the most important reason is that mathematics also had not yet been developed.

The emphasis is not upon the defects of the system, but on the limitations because, ironically, the defects did not hinder the growth of logic. It may also be true that had the defects been detected very early, situation would not have been much different because time was not ripe for take-off of symbolic logic. One serious limitation of Aristotelian system is its narrow conception of proposition. He restricted it to subject-predicate form. Though class-relation is implicit in this theory of syllogism, Aristotle ignored it. There is little wonder that Aristotle did not think of any other relations. Consider these two examples:

All men are mortal.
All mortal beings are imperfect

∴ All men are imperfect.

Bangalore is to the east of Mangalore.
Madras is to the east of Bangalore.

∴Madras is to the east of Mangalore.

Both these arguments are valid in virtue of transitive relation. Aristotle recognized only the first example as valid and what is surprising is that he considered only the first type as an argument. The result is that most of the mathematical statements ceased to be propositions in his analysis. His narrow outlook eliminated any possibility of logic and mathematics interacting. Consequently, considerable types of arguments with much complicated structure fall outside the limits of Aristotelian logic and hence remain unexamined. Medieval logic, in spite of remarkable contributions to logic, did not take logic a step ahead because whatever research was done was only an in-house work, i.e., work within the system. What was required was transition from one system to another.

In what sense modern logic makes progress over Aristotelian logic? It is very important to answer this question. Modern logic did not supersede Aristotelian logic in the sense in which an amendment to constitution results in one act replacing another. Modern logic neither superseded nor succeeded Aristotelian logic. It only extended the boundaries of the system. Existing rules remained not only acceptable but also were augmented by new set of rules. Later we will learn that among nine rules of inference, six are from Aristotelian logic. And simple conversion and observation were retained but given 'extended meaning' in terms of the rules of commutation and double negation respectively. Meaning was extended because logic and mathematics mutually made inroads into one another's territory. In a similar fashion, the use of variables also underwent a change. While Aristotle used variables only to represent terms, modern logic extended the use to propositions as well. This inclusion had far reaching consequences. Lastly, quantification, which was introduced during medieval age, was further improvised.

The foregoing discussion should make one point clear. The tools used to test arguments or to construct arguments by Aristotelian system are insufficient. Modern logic further augmented the tools not only in number but also in variety. It should be remembered that the sky is the limit to improve and add.

Before we enter the modern era, one interesting question must be considered. How should we explain the relation between logic and mathematics? Two philosophers have differently described this relation. Raymond Wilder says that for Peano and his followers 'logic was the servant of mathematics'. Wilder put it in a more respectable and acceptable form, in connection with Frege's philosophy of mathematics, 'dependence (of mathematics) on logic… was more like that of child to parent than servant to master. Basson and O'connor have echoed more or less similar views while relating classical logic to modern logic. It is like embryo related to adult.

## 1.4 HISTORY AND UTILITY OF SYMBOLIC LOGIC

At this stage, two aspects must be made clear. Modern logic is also called symbolic logic because symbols replaced words to a great extent. Second, symbolic logic and mathematics do not stand sundered; so much so, modern logic is also called mathematical logic, which A.N. Prior terms 'loosely called.' However, Prior's remark has to be taken with a pinch of salt. Very soon, we realize that almost all people, whose names are associated with symbolic logic, are basically mathematicians. And at some stage it becomes extremely difficult to separate logic from mathematics and, if attempted, it will be an exercise in futility. However, a definite limitation must be considered. When we talk of mathematics we talk of pure mathematics only. So when we deal with history of a symbolic logic we deal with the history of pure mathematics.

Where exactly does symbolic logic score over classical logic? Language is, generally, ambiguous. It is so for two reasons. In the first place, a significant number of words are equivocal and secondly, many times the construction of sentences and their juxtaposition are misleading so much so they convey meaning very different from what the speaker or author intends. Replacement of words by symbols and application of logical syntax different from grammatical syntax completely eliminates ambiguity. The meaning of logical syntax becomes clear in due course when sentences are represented by symbols. It is possible to test the validity of arguments only when the statements are unambiguous. Further, use of symbols saves time and effort required to test the validity of arguments.

## 1.5 THE RISE OF SYMBOLIC LOGIC

Generally, bibliography of symbolic logic compiled by Alonzo Church is reckoned as authentic to determine the beginning of symbolic logic. In the year 1666, Leibniz published (or wrote) a thesis on a 'Theory of Combinations titled '*Dissertatio de Arte Combinatoria*.' It is said that the beginning of symbolic logic coincides with this work. If so, Chrysippus has to be heralded as the forerunner of symbolic logic because according to records long before Leibniz he showed some interest in Combinations. So he must have done some work on Combinations, which was, further, followed up by some logicians in the thirteenth century. In brief, let us describe the subject-matter of Combinations. Leibniz was more concerned with such issues as semantic interpretations of logical formulas. One example may clarify semantic consideration or considerations which engaged Leibniz. What does the statement 'All men are mortal' mean? Does it mean that every member of the class of men is also a member of the class of mortal beings? Or does it mean that every man possesses the attribute of being a mortal? Or does it mean that the attribute of 'being man' includes the 'attribute of being mortal'. In other words, the focus of this consideration is on the choice between extensional approach and intentional approach. Class-membership issue is extensional whereas attribute-inclusion or attribute – exclusion is intentional.

Another notable contribution of Leibniz was his work on logical algebra or logical calculus, which consists of several experimental sorts of studies. Some laws, which are features of his study, are laws of identity and explicit statement of transitive relation, which made Aristotelian syllogism significant. Consider these two rules:
a b is a
ab is b
These rules become intelligible when we substitute terms for a & b. suppose that a = intelligent; b = man

1) Intelligent man is a man
2) Intelligent man is intelligent

Likewise consider another rule:
if a is b and a is c then a is bc.
Again substitute of, b and c, a = Indian, b = Asian, c = Hindu. Then 3 becomes
If Indian is an Asian and Indian is a Hindu, then Indian is an Asian Hindu.

An important requirement of logical algebra is that substitution must be possible; this particular relation was explicitly recognized by Leibniz.

In the 18th century two mathematician, Euler and Lambert contributed to the development of logic. While Euler is known for geometrical representation of propositions through his circles, Lambert developed logical calculus on intensional lines. For example, if a and b are two concepts, then a + b becomes a complex concept and ab stands for conceptual element common to a and b. What applies to class membership applies also to attributes. Bolzano is another logician who contributed to logic in the 19th century. He regarded terms and propositions as fundamental constituents of logic. He is known for an extraordinary approach to the logical semantics of language. In this context, he regarded propositions as having universal application when certain conditions are satisfied and as universally inapplicable under certain other conditions and as consistent under certain other conditions. Bolzano in fact, modified Kant's definition of 'analytic judgment' using this particular criterion. Another important contribution of Bolzano was his conception of probability. He introduced some modifications into Laplace's conception of probability, which was widely held during his time. Laplace defined probability as equipossible while determining the probability value when only two possibilities are available as in the case of tossing of the coin. In fact, Bolzano's modification avoids this particular element. This is crucial because 'equipossible' involves circularity. By avoiding this term, Bolzano could avoid circularity, which was inherent in Laplace's theory.

In 1847 two mathematicians, de Morgan and George Boole published 'Formal Logic' and 'The Mathematical Analysis of Logic respectively. Symbolic logic actually took off from this point of time. De Morgan gave to the world of logic now famous notion of complement which was later exploited by John Venn to geometrically represent distribution of terms and test syllogistic arguments. De Morgan showed that if there are two classes, then there are four product classes and Jevons showed that if there are three classes, then there are eight product classes. So generalizing this relation, we can say that the relation between the number of classes and the number of product classes is given by the formula, $n = 2^x$. Where, 'n' stands for the number of product-classes and x stands for the number of terms. This formula is only indicative of the type of relation, which holds good between classes (or sets) and product classes because there is no syllogism with more than three terms and no proposition (in traditional sense) has more than two terms. He also gave a formula known as de Morgan's law to write the contradiction for disjunctive and conjunctive propositions.

Boole's contribution to the rise of symbolic logic far exceeded that of any other logicians considered so far. He conceived the idea that the laws of algebra do not stand in need of any interpretation. This idea led Boole to describe these laws as calculus of classes in extension. In 1854 he published another work 'An Investigation of the Laws of Thought!' It is in this work that the germs of the 20th century symbolic logic can be traced.

While Lambert invented union of concepts on intensional analysis. Boole invented union of sets on extensional basis. He used '1' to designate the universe. Following de Morgan, Boole called it the universe of discourse. He introduced the following laws, which play crucial role in mathematical logic.

1 Union of any set and universal set is a universal set. Let X be a set. Then 1+X = 1
2 Product of a universal set and any non-null set X is X itself.
3 Product of null-set and any non-null set (universal set included) is a null-set itself.
If X is a non-null set, the 1 – X is its complementary.
5 It is self-evident that product of any non-null set and its complementary is a null-set.

5 Stands for Boole's definition of contradiction. He also showed that if X, Y, Z,…etc. stand for non- null sets, then all laws of algebra hold good. Most important among them are what are known as distributive and commutative laws. For the sake of brevity, these laws are stated as follows:

1 Distributive Law: a(b+c) = ab + ac
2 Commutative Law: ab= ba
or a+b=b+a

Using the concept of complementary class, Boole also showed that 'A, E, I and O' of traditional logic can be reinterpreted. His suggestion was geometrically represented by Venn.
In this interpretation, Boole actually considered what is called class logic, which later became the cornerstone of set theory. In logic, there is another topic called calculus of propositions. Boole integrated these two and defined the truth-value of what are called compound propositions which also consist of variables. While in the first interpretation the variables represent the sets or terms, in the second interpretation they represent the propositions. Consequently, products of classes, here, become conjunction and union or addition of classes becomes disjunction. Complement of a set becomes negation of a proposition.

Boolean analysis of logic is also called Boolean algebra for two reasons. In the first place, he freely used variables to explain various aspects of logic. Extensive use of variables characterizes algebra. Secondly, he defined all four operations of algebra; addition, multiplication, subtraction and division and extended the same to logic.

Venn's contribution to logic was partially mentioned earlier. Therefore the remaining part requires to be mentioned. Venn is well-known for making qualitative distinction, in addition to traditionally held quantitative distinction between universal and existential (particular) which has far reaching consequences. The distinction is that while universal proposition (in modern logic universal quantifier) denies the existence of membership in a class, existential quantifier affirms the same. Secondly, a large number of deductive inferences became invalid as a result of this description. The irony is that in this situation, progress is marked not by augmentation but by depletion in the number of inferences.

There were certain anomalies in Boolean system. Consider two identical sets, say X and Y where every member of X is a member of Y and every member of Y is a member of X; for example, the class of bachelors and the class of unmarried men. The product class should yield X Y. Since Y = X, $XY=X^2$ or $Y^2$. In algebra it makes sense, but surely not in logic. Similarly X+Y, the union of two sets ought to become 2X. Again, it holds good in algebra but not in logic. Jevons, a student of de Morgan, succeeded in eliminating these anomalies; according to his interpretation, the union of two identical sets does not double the strength, say from n to 2n. The reason is simple; every member is present in both the sets. We cannot count one individual as two just because he or it is present in two sets simultaneously. The same reasoning applies to product of identical sets. If there are 100 bachelors and 100 unmarried men then the product of these two sets does not produce $100^2$ = 10,000 bachelors who are also unmarried men, but 100 only. C.S. Peirce resolved this anomaly in a different way. He identified logical addition with *inclusive or* instead of *exclusive or* (either p or q but not both is an example for exclusive or and either p or q or both is an example for inclusive or).

Peirce introduced a symbol $\supset$ for class inclusion. He strangely argued that there is no difference between a proposition and inference or implication. In the ultimate analysis only implication survives. Secondly, all implications have quantifiers, which may be explicit or implicit. While Peirce thought that implication is the primary constituent of logic, at a later stage, there were attempts to eliminate implication and retain only negation and conjunction. While introducing symbols in a set of formulas Peirce was driven by a definite motive. He believed that symbols should resemble what they represent say thoughts. To achieve his aim, Peirce used, what he called, 'existential graphs'. They were not graphs in geometrical sense. He regarded parentheses themselves as graphs. For example, 'if p, then q'was represented graphically, by Peirce by using parentheses. He inserted p and q within parentheses and represented as (p (q)).

Christine Ladd Franklin invented a new technique of testing syllogism called antilogism or inconsistent triad. In addition to, Venn's diagram, antilogism also eliminated weakened and strengthened moods on the ground that particular propositions cannot be deduced from universal propositions only.

Gottlob Frege is one of the pioneers, who gave a new dimension to mathematical logic. In 1879 'Begriffsschrift' the first of his most important works was published followed by Die Grudlagan der Arithamatik in 1884. His first work dealt with proper symbolization with the help of rules of quantification. His intention was to codify logical principles used in mathematical reasoning like substitution, modus ponens, etc. In this work he introduced the notion of function, which was later renamed as propositional function. He also introduced a system of basic formulas for propositions in terms of implication and negation. In his second work, Frege made the most crucial attempt to trace the roots of mathematics to logic. He himself regarded arithmetic as simply a development of logic. Consequently, every proposition of arithmetic became merely a law of logic. History has recorded that Frege's thesis would not have got what it deserved but for Russell's discovery of Frege. Hence the relation between arithmetic and logic is known as Frege-Russell thesis.

It is said that modern logic began with Frege. It means that in one sense the history of symbolic logic stops before Frege. Whatever development that took place after Frege's period characterize contemporary logic. Even in this period, there were remarkable changes with new theses being presented regularly. Giuseppe Peano tried to establish the relation between logic and mathematics in a slightly different manner. Instead of tracing the roots of mathematics to logic, Peano tried to express mathematical methods in a different form similar to that of logical calculus. For example, the successor of 'a' was designated by the symbol 'a+'; also in addition to the symbol $\supset$ he introduced another symbol $\in$. This shows that implication or class inclusion ($\supset$) is distinct from 'element of' or 'belongs to'. In Peano's system there is no interpretation of any symbol and hence mathematics becomes a formal system.

In the beginning of the 20th century Zermelo proposed his theory of sets known as Axiomatic Set theory. He intended his theory to be free from contradictions. He regarded it as well ordered because it was axiomatized. His claim was totally rejected by Poincare. Perhaps only two mathematicians disputed the theory that mathematics has its foundations in logic. Opposition to this approach developed first in the 19th century. Kronecker, a professor of mathematics at the University of Berlin in 1850s, was the first mathematician to oppose this dominant trend. He disagreed with Cantor's theory of sets which included the concept of infinity. Kronecker went to the extent of arguing that integers are made by God, but everything else is the work of man. After Kronecker, it was Poincare who believed that mathematics does not have its base in logic. His main thesis is that in the first place, mathematical induction cannot be reduced to logic; secondly, according to him, even mathematics proceeds from particular to universal only; a clear opposition to deductive logic.

## 1.6 THE AGE OF *PRINCIPIA MATHEMATICA*(PM)

In 1910 Bertrand Russell in association with A.N. Whitehead published *Principia Mathematica*. What was referred to as the Frege-Russel thesis in the previous section found exposition in this work. Only a few aspects of this great work can be dealt here. The principal thesis remains the same, that mathematics is an extension of logic. Jevons, earlier, remarked that 'algebra' is nothing but highly developed logic' to which Frege added: 'inferences….. are based on general laws of logic.' Frege was actually referring to mathematical induction. In the preface itself the authors admitted that ‘thanks to Peano and his followers symbolic logic… acquired the technical and the logical comprehensiveness that are essential to a mathematical instrument’. Clearly, the new age mathematicians bypass Poincare and Kronecker in this regard.

PM makes a clear distinction between proposition and propositional function. While variables constitute propositional function, substitutions to variables constitute propositions. The former is neither true nor false. But the latter is either true or false. For example, X is the husband of Y is neither true nor false. But Rama (X) is the husband of Sita (Y) is true.

A key logical term, which finds place in PM is material implication. Russell and Whitehead used ‘$\supset$’ to designate implication. Material implication is defined as follows:

$p \supset q \equiv \sim p \vee q$

Truth-values were assigned by PM as follows. Both p and q can be true together, or when p is false, q may be false or true. But when p is true q cannot be false. Implication, therefore, does not imply necessary connection. To distinguish implication from prohibited possibility Russell and Whitehead used material implication instead of mere ‘implication’.

This particular definition of material implication has a very important consequence. ‘Necessary relation’ was an unwanted metaphysical baggage, which was overthrown by Hume. But there was no way of interpreting implication in the absence of necessary relation. Fixation of truth-value by PM made a distinct advance in this case. And it is precisely this type of implication that is used in mathematics. Consider a very familiar example, ‘If ABC is a plane triangle, then the sum of the three angles equals two right angles’. That there is no plane triangle at all does not affect the relation because even when the antecedent is false the consequent can continue to be true. Hence it comes to mean that a true premise can imply only true conclusion whereas a false premise can imply either true or false conclusion.

PM includes five axioms (Russell and Whitehead use the word ‘principle’), which can be regarded as primitive logical truths. They are follows:

| | | |
|---|---|---|
| 1 | Tautology | (Taut) |
| 2 | Addition | (Add) |
| 3 | Permutation | (Perm) |
| 4 | Association | (Assoc) |
| 5 | Summation | (Sum) |

Example provided here is taken from the text itself. The authors in all these cases use the symbol I- which is read ‘it is asserted that’ or it is true that’ and the dots after assertion I– sign indicate range. ‘v’ is read ‘or’ and ‘$\supset$ is read ‘if...then’.

| | | |
|---|---|---|
| Taut: | I-: $p \vee p. \supset .P$ | It is true that p or p implies p. |
| Add: | I-: $q. \supset . p \vee q$ | It is true that if q, then p or q. |
| Perm: | I-: $p \vee q. \supset .q \vee p$ | If p or q, then q or p. |
| Assoc: | I-: $p \vee (q \vee r). \supset. q \vee (p \vee r)$ | If p or q or r, then q or p or r. |
| Sum: | I- $:q \supset r. \supset: p \vee q. \supset .p \vee r$ | If q implies r, then p or q implies p or r. |

For 'Add' the example is 'if today is Wednesday (q), then today is either Tuesday or Wednesday. The examples can be constructed on similar lines for other axioms. For perm, the example read as follows; if today is Wednesday or Tuesday, then today is Tuesday or Wednesday. In all cases, the sentences are preceded by 'it is true that'. The colleen immediately after the assertion sign indicates range, but the dots which follow or precede variables are only customary.

PM also includes equivalence relation, which explains the equivalence of the law of the Excluded Middle and the Law of contradiction. In the beginning of the summary of *3 the authors say that 'it is false that either p is false or q is false, which is obviously true when and only when p and q are both true. Symbolically,

$$p.q = \ . \sim (\sim p \vee \sim q)$$

*Reductio ad absurdum* is one method accepted by mathematics. It means that the contradiction of what has to be proved is assumed to be true and then the conclusion contradicting the assumption is deduced. This contradiction shows that the assumption is false in which case its contradiction must be true. This is again a primitive logical truth. The principle of double negative is another, which can be easily derived from the law of the Excluded Middle.

David Hilbert contributed to the development of logic which led to the birth of what is known as metamathematics. His theory of mathematics is known as formalist theory of mathematics. This theory of mathematics makes a distinction between sequence and statement. It asserts that a sequence is neither true nor false. This distinction corresponds to the one made in classical logic between a sentence and a proposition. An important aspect of metamathematics is its axiomatic approach. A system, be it mathematics or anything else, can be formalized only when axiomatic method is followed. A system is said to be formalized or axiomatized only when all propositions in the system stand in a definite logical relation. Consistency is one such relation. Therefore, a consistent system, in Hilbert's analysis is an axiomatized system.

A distinguishing mark of Hilbert's analysis is his 'discovery' of 'ideal limit'. From the days of Cantor and Weirstrass who introduced the concept of 'infinity' or 'transfinite' the concept of ideal limit engaged the attention of mathematicians. While elementary number theory could be empirically interpreted, infinity could not be interpreted in that manner. So Hilbert chose to regard transfinite as limit.

There should not be break in history – circuit. Therefore another contribution of Hilbert secures a place in our discussion. Hilbert embarked upon his project to defend classical mathematics from one theory of mathematics known as intuitionism spearheaded by the Dutch mathematician Jan Brouwer, according to whom mathematics is not a system of formulas but is a sort of abstract activity, which abstracts the concept of 'numberness.' By any standard, 'intuitionist mathematics ceases to be a logical enterprise, but confines itself to the narrow domains of psychological activity at best and some sort of esoteric activity at worst.

Following the tradition of PM, Emil Post presented the method of truth-tables published as 'Introduction to a General Theory of Propositions' in the American Journal of Mathematics in 1921. In this paper, Post included not only classical logic, which allowed only two values but a system allowing many values. In the same year Wittgenstein's *Tractus logico-Philosophicus* was published, which also included this technique. Wittgenstein held the view that mathematics is nothing but a bundle of tautologies. While this is the view of earlier Wittgenstein, in later Wittgenstein the conception of mathematics underwent dramatic change. In 'Remarks on the Foundations of Mathematics' Wittgenstein argues that both logic and mathematics form parts of language games. At this point of time he became a conventionalist and argued that mathematical propositions are immune to falsification. This position of Wittgenstein is much closer to intuitionism than to anything else.

Rudolph Carnap's contribution to symbolic logic consists in the extension of the same to epistemology and philosophy of science. He argued that all meaningful sentences belong to the language of science. He followed what is called the 'principle of tolerance' with which any form of expression could be defended if sufficient logical rules are there to determine the use of such expression. Under the influence of Alfred Tarski, he included such notions as truth and meaning in his analysis.

Kurt Goedel is another important philosopher of mathematics. He was concerned with intuitionistic and classical mathematics equally. He is widely known for his famous '*Incompleteness Theorem'*. He showed that it is impossible to prove consistency of certain formulations of arithmetic by methods which are internal to the system. He showed that what is provable in classical mathematics is also provable in intuitionist mathematics. The only requirement is that what has to be proved must be properly interpreted.

Alonso Church is a noted historian of symbolic logic. Logicians and mathematicians alike are interested in questions related to the decidability of logical and mathematical theories. His main thesis is that there is no general technique to determine or discover the truth or proof of any proposition in arithmetic. In this respect, Church stands opposed to Hilbert who argued that classical mathematics is a consistent system. W.V.O Quine and Curry are two other prominent personalities. While Quine is known for his contribution to the development of set theory, Harkell B. Curry's name is associated with a new branch of logic called 'Combinatory Logic'. It had its birth in H.M.Shaffer's discovery of 'stroke' symbol (I) with which all sentential connectivity could be interpreted. This was extended by Moses Schonfinkel to quantifiers also. Stroke symbol was introduced to simplify the use of symbols and subsequently Schonfinkel extended it to eliminate variables. Curry proceeded further with Schonfinkel's works with set of operations different from stroke symbol. He introduced what is called the theory of λ – conversion (λ is read 'lamda'), where λ is known as binary operation. Church used this operation to analyze formal systems to which variables belong and to which arbitrary objects can be substituted. Here objects mean the functions in which they stand for arguments. It means that a variable in a system is substituted by an argument. λ – conversion is a theory proposed by Church in connection with such substitutions.

In short, symbolic logic is a system of algebraic combination and mechanical substitution of symbols for the purpose of inference. It is the study of symbolic abstractions that captures the formal features of logical inference. C.I. Lewis observes the following characteristics for symbolic logic: the use of ideograms (i.e., signs that stand directly for concepts) instead of phonograms (signs that depict sounds first and indirectly concepts); deductive method and use of variable having definite range of significance. It has mainly two parts: truth-functional or propositional or sentential logic and predicate logic. The former is a formal system in which propositions can be formed by combining simple propositions using sentential connectives, and a system of formal proof in determining the validity of arguments. Predicate logic provides an account of quantifiers in the symbolization of arguments and laws for the determination of their validity.

**Check Your Progress**

**Note:** a) Use the space provided for your answer.
b) Check your answers with those provided at the end of the unit.

1. Examine Boole's contribution to modern logic.
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2. Examine the role played by PM in the 20th century logic.
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3.Contrast Hilbert's and Goedel's views on proofs in mathematics.
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4. What is the significance of Shaffer's and Schonfinkel's studies? Explain.
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## 1.7 LET US SUM UP

Logic has its roots in Greek civilization. Aristotle systematized the technique of thinking. During medieval ages, lot of research work was undertaken within the limits of Aristotelian system. Modern logic took its birth with Leibniz' work '*Dissertatio de Arte Combinatoria*'. Boole's works provided impetus to the growth of symbolic logic. Contemporary symbolic logic begins with de Morgan. Initially, Frege and Russell and later, Russell and Whitehead heralded a new era in symbolic logic. Combinatory logic has its beginning in H.M. Shaffer's work which was later developed by Haskell B. Curry. Today logic and mathematics have become two faces of the same coin.

## 1.8 KEY WORDS

**Theorem:** In mathematics, a theorem is a statement proved on the basis of previously accepted or established statements such as axioms.

## 1.9 FURTHER READINGS AND REFERENCES

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## 1.10 ANSWERS TO CHECK YOUR PRGRESSS

1 Boole's contribution to the rise of symbolic logic far exceeded that of any other logicians considered so far. He conceived the idea that the laws of algebra do not stand in need of any interpretation. This idea led Boole to describe these laws as calculus of classes in extension. In 1854 he published another work 'An Investigation of the Laws of Thought!' It is in this work that the germs of the 20th century symbolic logic can be traced.

2 The publication of Principia Mathematica by Russell and Whitehead heralded a new era in the history of mathematics and logic. In this work they established that logic is the foundation of mathematics. The term implication acquired a new meaning when new rules of inference were evolved. These rules of inference forced logicians to distinguish implication from entailment. Also this work influenced Emil Post to present the methods of truth-table which is the backbone of mathematical logic. The earlier Wittgenstein was also partly influenced by this work.

3. David Hilbert contributed to the development of logic which led to the birth of what is known as metamathematics. His theory of mathematics is known as formalist theory of mathematics. This theory of mathematics makes a distinction between sequence and statement. It asserts that a sequence is neither true nor false. This distinction corresponds to the one made in classical logic between a sentence and a proposition. An important aspect of metamathematics is its axiomatic approach. A system, be it mathematics or anything else, can be formalized only when axiomatic method is followed. A system is said to be formalized or axiomatized only when all propositions in the system stand in a definite logical relation. Consistency is one such relation. Therefore a consistent system, in Hilbert's analysis is an axiomatized system. Kurt Godel is another important philosopher of mathematics. He was concerned with intuitionistic and classical mathematics equally. He is widely known for his famous '*Incompleteness Theorem'*. He showed that it is impossible to prove consistency of certain formulations of arithmetic by methods which are internal to the system. He showed that what is provable in classical mathematics is also provable in intuitionist mathematics. The only requirement is that what has to be proved must be properly interpreted.

4 Combinatory logic had its birth in H.M.Shaffer's discovery of 'stroke' symbol (I) with which all sentential connectivity could be interpreted. This was extended by Moses Schonfinkel to quantifiers also. Stroke symbol was introduced to simplify the use of symbols and subsequently Schonfinkel extended it to eliminate variables.

## UNIT 2 COMPOUND STATEMENTS AND THEIR TRUTH-VALUES

**Contents**



## 2.0 OBJECTIVES

After you grasp the contents of this unit you should be in a position to:

- analyse any compound proposition to determine its truth-value.
- realise that always symbolic representation of statements helps better understanding than verbal representation which is not only more complicated in structure but also ambiguous.
- understand that a compound proposition may be highly complicated as far as structure is concerned, but it does not affect the technique of determining the truth-value.
- determine the width of spectrum of compound proposition and simple form of compound from the complicated form of compound proposition.

## 2.1 INTRODUCTION

In this unit an attempt is being made to project the structure of and variety in proposition in a new perspective. Secondly, two shades of meaning of compound proposition will be distinguished in order to accommodate one type of statements, which looks like simple. A clear definition of truth-function is attempted by considering two parameters simultaneously.

## 2.2 SIMPLE AND COMPOUND STATEMENTS

In this unit, we consider two kinds of statements; simple and compound. This kind of distinction is similar to grammatical distinction. However, there is a sharp difference. A compound statement in grammatical sense is independent of its components as far as its truth-value is concerned. However, in logical sense the truth or falsity of compound proposition depends upon the truth or falsity of its components. Simple proposition does not need any definition. It consists of only one sentence in grammatical sense. Compound statement, on the other hand, consists of two or more than two 'statements'. The last word should be carefully observed. It just says 'statements'. In other words, the components of a compound statement may be simple or themselves compound. Though the distinction per se is too a simple, statements may be deceptive. Consider the following examples:

1 Grass is green.

2 Einstein is a physicist and Lorenz was his professor.

3 Descartes is a philosopher and mathematician.

It is easy to conclude that the first statement is simple and the second statement is compound. However, we should not be hasty in judging the third proposition. It only seems to be a simple proposition. In reality, it is a compound statement. It can be analysed as follows: Descartes is a philosopher and Descartes is a mathematician. In the language of predicate logic compound proposition can be understood as follows; if there are two predicates then there are two propositions. And if there are three predicates, then there are three propositions and so on.

## 2.3 SENTENTIAL CONNECTIVES

A compound proposition can be generated in several ways. Classical logic says that a proposition is generated when subject and predicate terms are conjoined by copula. Likewise, modern logic says that a compound proposition is generated when two or more than two propositions are conjoined by what is known as sentential connective. There are five types of sentential connectives and therefore, there are five types of compound statements. 'And', 'if…then', 'or', 'not' and 'if and only if' (iff) are the connectives used to conjoin the statements. While providing descriptive account, connectives are shown, initially, in upper case letters for the sake of clarity only. Further, all letters printed in lower case below statements symbolise respective statements.

I) AND: 'AND' is one type of sentential connective. When two propositions are connected by this connective, a compound proposition is generated. This type of compound proposition is known as 'CONJUNCTIVE' proposition or we simply say 'CONJUNCTION'. Consider first simple propositions:

Water flows down hill.

The sun is bright.

It is very easy to form a conjunctive proposition; just place 'AND' between 'water flows downhill' and 'the sun is bright'. We get the statement

Water flows downhill AND the sun is bright.

p q

When we are doing symbolic logic, we hardly construct statements with words. Nor do we use 'AND' while writing a conjunctive proposition. Otherwise, it ceases to be symbolic logic. This connective is symbolized in two ways. The old style is '**.**' And the present style is 'Λ'. We will follow the latter. Now we will symbolize the proposition:

Water flows downhill: p
The sun is bright: q
The conjunction is as follows: (water flows downhill) and (the sun is bright).
p Λ q

p Λ q is the form of conjunction. When an argument is being tested propositions are symbolised in the following manner. p is replaced by W and q is replaced by S; therefore p Λ q is replaced by W Λ S. This change is useful when there are several statements. This particular classification applies equally to other compound propositions, which involve other sentential connectives.

II) IF….THEN: A compound proposition generated with this particular connective is known as 'IMPLICATIVE' proposition or simply 'IMPLICATION'. It is also called hypothetical. The latter, usage, however, is restricted only to classical logic. In order to obtain implicative proposition the first word 'if' is inserted in the very beginning of compound proposition; 'then' is inserted between two components. We will show the process of conjoining these statements with an example: 'There is no end to political turmoil'; 'Economic prosperity will be badly hit'. We obtain the following implicative proposition: 'IF there is no end to political turmoil, THEN economic prosperity will be badly hit.' We shall symbolize it as follows:

7 There is no end to political turmoil: p
8 Economic prosperity will be badly hit: q
9 If p, then q.; this is the form of implicative proposition.

Replace the form by symbols for propositions. We get
If T, then E.

Now we will take second step. The connective 'if..... then' also is symbolized. Again there are two ways of symbolizing the same. '⊃' and '=>'. We shall use only the latter; p => q. '⊃', which is read horse shoe, is not used now to show implication because this symbol is used in set theory to show class inclusion. In order to avoid ambiguity and confusion we represent implication with the symbol =>.

III) OR: When 'OR' connects two propositions we obtain DISJUNCTIVE proposition or simply DISJUNCTION. Some authors like Cohen and Nagel preferred to call it ALTERNATIVE proposition or simply ALTERNATION. At the outset, we should distinguish two senses in which this connective is often used. One is called 'inclusive' or and the second one is called 'exclusive' or. The process of obtaining disjunction is very simple. The connective 'OR' is placed between simple propositions. The resultant statement is a disjunctive one. Take these statements:

10 Reason is the true friend of mankind. p
11 Treason is the worst enemy of the state. q

With these two statements we obtain the required disjunctive statement:

12 'Reason is the true friend of mankind OR Treason is the worst enemy of the state'. When it is symbolized, it becomes p or q. The connective 'OR' is symbolized by using the symbol 'v'. This symbol is called Wedge. p or q becomes p v q. This particular statement is an example for 'inclusive' OR. It is called inclusive because the statement also includes third possibility. Accordingly, it can be further extended in the following manner:

13 'Reason is the true friend of mankind or treason is the worst enemy of the state' or both.

The last word 'both' is the extended part of original compound statement. This is third possibility, which cannot be logically ruled out. If third possibility is admissible in any disjunctive proposition, then 'OR' becomes inclusive. There are cases when third possibility is not admissible. Consider these two statements:

14'Rich people are generous or greedy.'
It does not admit further extension. It does not make sense to say that

15 'Rich people are generous or greedy or both generous and greedy.'
Since the extended part is inadmissible in this example 'OR' is regarded as exclusive or. When disjunction consists of exclusive or, the proposition is symbolized as

p v q

At this juncture a clarification is necessary. When is 'OR' inclusive and when is it exclusive? There is no law of logic as such which stipulates the conditions under which 'OR' becomes inclusive and conditions under which 'OR' becomes exclusive. We have to depend upon the 'meaning' of certain terms employed in the construction of statements. Consider propositions 10 and 11. We admit that these two statements do not exclude each other based on what these statements 'really' mean. However the same is not the case with propositions 14. The terms 'greedy' and 'generous' mean so differently that they both 'cannot' be the attributes of the very same class or individual. In other words, if rich people are greedy surely some other class of people can be generous and vice versa. Hence meaning alone can be our guide in determining whether 'or' is inclusive or exclusive.

Generally, disjunction is expressed in terms of 'EITHER ... OR'. There is no harm in omitting the former. Both usages are admissible.

IV) NOT: In modern logic, when the connective NOT is appended to the given propositions, it becomes a compound proposition. However, grammar does not allow it. Therefore we have to treat this as a special case within the structure of modern logic. We obtain 'NEGATION' when NOT is used. This is another kind of compound proposition in strictly logical sense because the use of this word alters the truth-value of the given proposition. The connective NOT is appended to the given propositions in several ways. Negation may begin with expressions like "It is NOT the case that… … …" or "it is NOT true that… … …" Consider this example:

16 The sun rises in the east. - p
Now this statement is negated and expressed in three different ways.

17 It is NOT the case that the sun rises in the east. - NOT p
17a It is NOT true that the sun rises in the east. - NOT p
17b The sun does NOT rise in the east. - NOT p

It must be noted that all these three statements exactly mean the same and all of them negate the statement 16. Now we will symbolize the statement, using symbol for negation, '⌐'

16 p
17 ⌐ p

'Not' was symbolized earlier in a different way. The symbol '~' was used earlier to denote negation. This is read curl or tilde. Russell and others used this symbol.

V IF AND ONLY IF: When this connective is used we obtain 'BICONDITION'. We will insert this connective between two statements to obtain 'BICONDITIONAL' proposition. Consider these two examples:

18 Mr. A is a bachelor. - p
19 Mr. A is an unmarried male. - q

Now connect 18 and 19 using the given connective.
Mr. A is a bachelor IF AND ONLY IF Mr. A is an unmarried male.
This connective is symbolized in this manner '<=>'. BICONDITIONAL proposition is represented as follows; p <=> q. Negation (⌐) and biconditional are (p <=>q) special kinds of compound proposition. This will become clear in the next section.

**Check Your Progress I**

**Note:** a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1) Distinguish 'compound' in grammatical sense from 'compound' in logical sense.

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2) Bring out the difference and similarity with respect to copula and sentential connective.

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## 2.4 COMPOUND PROPOSITIONS AND THEIR TRUTH-VALUES

Classical logic stipulates that any proposition is either true or false. The truth-value of a true proposition is TRUE and the truth-value of a false proposition is FALSE. Truth-value refers to the designating of a statement either as true or false. Likewise, any compound proposition is either true or false. There is a technique of determining the truth-value of compound proposition. In effect the truth-value of a compound proposition is a function of the truth-value of its constituent or component statements. Barring a few cases, which are exceptions, in all other cases this particular specification applies to compound proposition. Therefore it is very important to distinguish these two kinds of compound proposition. It is distinguished as follows: **'A compound proposition is said to be truth- functionally compound if and only if its truth-value is a function of the truth-value of its components'**. In other words, truth-function is a compound statement whose truth-value is completely determined by the truth-values of its components. Logic which deals with truth-functional compound statements is called truth–functional logic: this is the part that we are presently studying.

The construction of truth-table (which is the list that shows the various values a truth-function may assume) is a technique adopted in order to determine the truth-value of compound propositions. It is interesting to learn that even when the propositions remain the same, different types of compound propositions exhibit different truth-values because sentential connectives change from one compound to another compound. This clearly shows that the sentential connective plays a crucial role in determining the truth-value of a compound proposition. *Therefore the truth-value of a compound proposition is determined by the truth-values of components and also the sentential connective used.* In order to drive home this paint, let us retain the same set of statements, which form parts of compound proposition, but at the same time obtain different results in terms of truth-values by using different sentential connectives.

21 The stars are self-luminous. - p

22 Glass is fragile. - q

Let us construct truth-tables to determine the truth-values of compound propositions (As usual '1' stands for 'True' and '0' stands for 'False'). Generally, no justification for determination of truth-value is called for. They are to be treated as the truth-conditions of respective compound propositions.

I) IMPLICATION:

An implicative proposition is false only under one circumstance, i.e., *when the antecedent is true and the consequent is false*. It means that false conclusion does not follow from true premise and under all other circumstances it is true. In the case of implication antecedent is the premise and consequent is the conclusion. Let us illustrate it in the form of a table.

**Table1:**

| | **p** | **q** | **p => q** |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 2 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 |
| 4 | 0 | 0 | 1 |

From this table one aspect becomes clear; a false premise implies any conclusion (whether true or false). It also means that a true conclusion follows from any premise. This is admissible because there is no necessary relation between the premise and the conclusion as pointed out earlier. (See 1.4) Implication as understood in logic is very different from common man's perception. This is exactly what Russell meant when he introduced the term 'material implication'.

Let us consider implication in verbal form. The statement 'If the stars are self-luminous, then glass is fragile' is false only when it is true that the stars are self-luminous and it is not the case that glass is fragile; and under all other circumstances it is true. This entire expression is hidden in Table 1. It is anybody's guess that Table 1 is more intelligible and understood with less effort than verbal form.

II) CONJUNCTION:

*A Conjunction is true if and only if both the conjunctions are true; otherwise, it is false*. Therefore, the truth-table for conjunction is as follows:

**Table: 2**

| | p | q | p Λ q |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 2 | 1 | 0 | 0 |
| 3 | 0 | 1 | 0 |
| 4 | 0 | 0 | 0 |

Conjunction corresponds to a familiar algebraic rule. When two positive numbers are added we will get sum. However, when a negative number is added to a positive number, we are only subtracting. And addition of two negative numbers also amounts to subtraction only. (-4+ (-4) = -8; and –8 < -4). Let us restate conjunction in verbal form:

i) The stars are self-luminous: 1
ii) Glass is fragile: 1

Conjunction:
1 The Stars are self-luminous and glass is fragile. 1
2 The Stars are self-luminous and glass is not fragile: 0
3 The Stars are not self-luminous and glass is fragile: 0
4 The Stars are not self luminous and glass not fragile: 0

III) DISJUNCTION:

*A disjunction is true when at least one of the disjuncts is true*. The condition of its truth-value can also be stated in this manner. *A distinction is false if and only if both the disjuncts are false*. Stated in this form, disjunction is just the inversion of conjunction. The truth-value for disjunction is as follows.

**Table: 3**

| | p | q | p v q |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 2 | 1 | 0 | 1 |
| 3 | 0 | 1 | 1 |
| 4 | 0 | 0 | 0 |

At a later stage we will have an opportunity to understand the significance of the way in which the truth-value conditions of disjunction and conjunction differ. For the time being, let us consider the verbal form of disjunction.

i) The Stars are self-luminous. 1
ii) Glass is fragile. 1

Disjunction:

| | | |
|---|---|---|
| 1 | The stars are self-luminous or glass is fragile. | 1 |
| 2 | The stars are self-luminous or glass is not fragile. | 1 |
| 3 | The stars are not self luminous or glass is fragile. | 1 |
| 4 | The stars are not self luminous or glass is not fragile. | 0 |

IV) NEGATION:

The simplest form of truth-functionally compound proposition is negation. In this case we have only two rows because there is only one proposition whereas in all other cases there are four rows because there are two propositions.

**Table: 4**

| | **p** | **⌐ p** |
|---|---|---|
| 1 | 1 | 0 |
| 2 | 0 | 1 |

If p stands for 'The stars are self-luminous', ⌐ p stands for 'The stars are not self luminous'. Therefore if 'it is true that the stars are self-luminous', then it is not true that the stars are not self-luminous'. And if it is not the case that the stars are self-luminous, then it is true that the stars are not self-luminous. Again, it is obvious that the verbal form is more complex than the truth-table. Since negation connects one proposition only, it is called unary whereas all other connectives are called binary since they connect two propositions.

V) BI-CONDITION:

*A biconditional proposition is true only when both the components have the same truth-value.* Otherwise, it is false. The truth-value of biconditional proposition is as follows:

**Table: 5**

| | **p** | **q** | **p <=>q** |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 2 | 1 | 0 | 0 |
| 3 | 0 | 1 | 0 |
| 4 | 0 | 0 | 1 |

Now let us consider verbal form for bicondition. 'The stars are self-luminous if and only if glass is fragile' is true when 'it is the case that the stars are self-luminous' and also 'it is the case that glass is fragile' or when 'it is not the case that the stars are self-luminous' and also it is not the case that glass is fragile'. Under remaining circumstances, it is false. In such cases the verbal form is as follows:

1 The stars are not self-luminous if and only if glass is fragile.
2 The stars are self-luminous if and only if glass is not fragile.

Again, let it be made clear that whether we say it is not the case that 'the stars are self-luminous' or we say that 'the stars are not self-luminous, there is no difference in intended meaning.

Negation and bicondition are unique for different reasons. Negation is unique because, though in grammatical sense, the statement 'the stars are not self-luminous' is a simple statement, modern logic regards it as a compound statement only because its truth-value depends upon the inclusion or exclusion of the connective 'not'. So what determines the compound nature of a proposition is not really the number of statements, but it is the truth-functional quality of proposition. In this connection it is worthwhile to refer to exceptions mentioned in the beginning of this section. While all truth- functional statements are compound, all compound statements are not truth- functional. In other words, in exceptional cases, the truth-value of components does not determine the truth-value of 'apparent' compound propositions. Consider these propositions, which, obviously, have this form.

23. If there is rise in the temperature, then there is rise in mercury level.
24. If India has to win the cricket match, then the gods must be crazy.

(23) and (24) differ in structure, which we generally, do not notice easily. In order to clearly understand the difference, let us break (23) and (24) to get their respective components.

23a There is rise in temperature.
23b There is rise in mercury level.
24a India has to win.
24b The gods must be crazy.

(23a) and (23b) are true or false together. But the same cannot be said about (24a) and (24b). They are, really, neither true nor false together. Therefore though (24) is a compound sentence, it is not truth-functionally compound. Therefore what is grammatically a compound statement may not be truth-functionally compound and vice-versa.

Bioconditional proposition is unique for another reason. Implication does not allow simple transposition of antecedent and consequent whereas biconditional proposition allows only simple transposition of components. Consider p => q and q => p respectively with the help of truth-table.

**Table: 6**

| | p | q | p =>q | q => p |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 0 | 0 | 1 |
| 3 | 0 | 1 | 1 | 0 |
| 4 | 0 | 0 | 1 | 1 |

From rows (2) and (3) it becomes clear that (p =>q) ≠ (q =>p). This is because the truth of implication does not allow simple transposition. However, the case of biconditional proposition is different. We should remember that many disputes can be settled with the help of truth-table.

**Table: 7**

| | p | q | p <=>q | q <=>p |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 0 | 0 | 0 |
| 3 | 0 | 1 | 0 | 0 |
| 4 | 0 | 0 | 1 | 1 |

From tables (6) and (7) it is clear that what allows or does not allow simple transposition is the truth-condition only. This particular characteristic can be brought out clearly only when bicondition is contrasted with implication.

The role played by sentential connectives in determining the truth-value of compound propositions vis-a-vis the truth-value of the components themselves is better understood when we compare the truth-table of all compound propositions. However, negation is not required for this purpose, since it does not have components.

**Table: 8**

| | p | q | p =>q | p v q | p Λ q | p<=>q |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 0 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 1 | 0 | 0 |
| 4 | 0 | 0 | 1 | 0 | 0 | 1 |

Assume that in all columns p is replaced by proposition 21 and q is replaced by proposition 22. It is impossible that the truth-value of the proposition components differ from one situation to another. The position is like this; even when the same set of propositions with determinate truth-values form the components of various compounds propositions, the truth-value of one compound proposition differs from the truth-value of any other compound propositions. Before we arrive at this conclusion, we must compare the truth-value of component propositions in all possible circumstance. Even if in one circumstance there is variation in the truth-value, our stand is vindicated. For example, in the table 8, the last two columns possess different truth-values only in the fourth row. Therefore it is clear that in spite of the fact the same set of propositions form components of different compound propositions, the truth-value varies from column to column because besides components, the sentential connective also determines the truth-value of given compound proposition. So the truth-value of a compound proposition is 'uniquely' determined by the truth-value of its components only with respect to that particular compound. However, if we have to explain variation from one column to another, then we also have to consider the role played by sentential connectives. The difference can be aptly summarized in this way; 'vertical variation in truth- value of a compound proposition is a function of the truth-value of components only, whereas horizontal variation is a function of sentential connective' only.

## 2.5 OTHER FORMS OF COMPOUND PROPOSITION

In the beginning of this unit, it was mentioned that the components of a compound propositions themselves can be compound propositions. We will consider a compound proposition with only three propositions because then we will have eight rows and if there are four propositions we will have sixteen rows. It is because, since any component takes two truth-vlaues (i.e., either true or false), addition of a component would double the number of rows: thus for one component, only two rows as in the case of negation; for two, four rows, as we have seen in other truth table; for three, eight rows; for four, sixteen; for five, thirty two rows, and so on. However, with three simple propositions several compound propositions can be constructed. Therefore it will adequately serve our purpose. The variables and statements are as follows:

25 Alcoholism is a vice. p
26 Courage is a virtue. q
27 Yoga heals diseases. r

Various compound propositions can be constructed out of these propositions. Some of them are considered.

28 (p=>q) Λ (⌐q v r)
29 (p =>q) v (p Λ q)
30 (q v r) => p
31 (q => r) v (p Λ r)

It should not be difficult to substitute statements of p, q and r. It is left as an exercise to the students to do the same. There is something more important to clarify.

Apart from the fact that the components of propositions 28 to 31 are themselves compound, there are parentheses also. The significance and necessity of parentheses can be easily understood, when compared with simple arithmetic. Compare these two expressions:

i). (5+7)10 = 300
ii). 5+7 x 10 = 75

(1) is false. It is not even possible to say whether ii) is false or not. Knowing whether a certain expression is true or false is not very significant. But arriving at a determinate expression is significant. This is what exactly parentheses achieve when used appropriately. If they are not used, then it will be a mistake in mathematics, language and logic.

Let us consider statement 28 which has four connectives and therefore there are four compound propositions. Though one truth-table is sufficient for our purpose, in order to gain better understanding, we shall split the table:

**Table: 9**

| | **p** | **q** | **p =>q** |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 2 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 |
| 4 | 0 | 0 | 1 |

**Table: 10**

| | **r** | **⌐q** | **⌐q v r** |
|---|---|---|---|
| 1 | 1 | 0 | 1 |
| 2 | 0 | 0 | 0 |
| 3 | 1 | 1 | 1 |
| 4 | 0 | 1 | 1 |

(r v ⌐q is the same as ⌐q v r.) To take next step let us assume that (p => q) is one component and ⌐q v r is another component. Let us transpose columns 3 of Table 9 and Table 10 to Table 11 to compute the result.

**Table: 11**

| **p** | **q** | **⌐q** | **r** | **p => q** | **⌐q v r** | **( p => q ) Λ (⌐ q v r )** |
|---|---|---|---|---|---|---|

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| 3 | 1 | 0 | 1 | 1 | 0 | 1 | 0 |
| 4 | 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 5 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 6 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 7 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 8 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |

Before closing this section one has to learn the method of constructing truth-tables; it is a very interesting part of the study of symbolic logic. Truth-tables are constructed for truth-functions having statement variables that are customarily counted from the middle part of the alphabet like p, q, r, s, … Accordingly, 'Bacon is a writer' is a statement in English; it can be symbolized as 'B'; it can be represented in a variable form as simply 'p'. Before beginning the work of constructing the truth-table we have fix the specific form of the given statement, determine the columns under which the truth-values are to be arranged and limit the number of rows in accordance with number of variables in the specific form of the statement. Let us work with a compound statement: (A=>B) Λ (⌐B v C). Its specific form is (p=>q) Λ (⌐q v r)
Its truth-table is just above (no. 11).

[The students are advised to construct truth-tables for the remaining combinations, which are relatively simple. In all cases the number of rows is 8. Since practice makes man perfect, the students are advised to substitute statements for variables in all cases.]

**Check Your Progress II**

**Note:** a) Use the space provided for your answer.
b) Check your answers with those provided at the end of the unit.

1) Define truth-functional logic.
……………………………………………………………………………………………
2) Distinguish between implication and bicondition.
……………………………………………………………………………………………
……………………………………………………………………………………………..

## 2.6 LET US SUM UP

Modern logic distinguishes two kinds of statements. All truth-functional propositions are compound. 'Grammatical' compound is different from 'logical' compound. Truth-functional compound is a function of sentential connective and truth-values of components. Negation is the simplest (simplest in grammatical sense) form of compound. There are five types of compound propositions, each distinguished by its own set of truth-values. The truth-values of one compound differ from that of the others at least on one occasion. Difference between implication and bicondition are notable. Components of compound proposition can themselves be compound. To have at least one compound within a compound, we need at least three propositions.

## 2.7 KEY WORDS

**Ambiguity**: When a word or a statement carries more than one legitimate meaning it is said to be ambiguous.

**Turmoil:** Turmoil is a state or condition of extreme confusion, agitation, or commotion.

**Main Connective**: The connective that determines the basic form of a statement is called main connective. For example, (A=>B) Λ (⌐B v C) is a conjunction whose left hand conjunct is an implication and whose right hand conjunct is a disjunction.

## 2.8 FURTHER READINGS AND REFERENCES


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## 2.9 ANSWERS TO CHECK YOUR PROGRESS

**Check Your Progress I**

1. A compound statement in grammatical sense is independent of its components as far as its truth-value is concerned. However, in logical sense the truth or falsity of compound proposition depends upon the truth or falsity of its components.

2. Both copula and sentential connective perform the function of linking two distinct units; copula links two terms whereas sentential connective links two statements which may be true or false. The number of sentential connectives is always one less than that of statements. The same connective may occur more than once in the given compound proposition. While copula does not determine the truth of combination, the latter determines the same.

**Check Your Progress II**

1 Logic which deals with truth-functional compound statements is called truth–functional logic.

2 Implication is false only when the antecedent is true and consequent is false and under all other instances it is true. Bicondition is true only when both the components have the same truth-value, i.e., both components must be true or false together.

## UNIT 3 SYLLOGISM

**Contents**



## 3.0 OBJECTIVES

In this unit an attempt is made:

- to introduce to you salient features of syllogism, which forms an important part of classical or Aristotelian Syllogism.
- to integrate traditional analysis with modern analysis. In doing so, some vital differences between these analyses are brought to the fore.

## 3.1 INTRODUCTION

Syllogism is the most important part of Aristotle's logic. It is a kind of mediate inference in which conclusion follows from two premises. We consider two kinds of syllogism, viz., conditional and unconditional. Further, under conditional, there are two divisions: mixed and pure. We can consider conditional syllogism at a later stage. In this unit, we shall confine ourselves to unconditional syllogism or categorical syllogism.

## 3.2 THE STRUCTURE OF CATEGORICAL SYLLOGISM

For the time being, let us assume that syllogism means valid categorical syllogism unless otherwise qualified. Syllogism consists of two premises and a conclusion. Thus, we have three prepositions and only three terms. An argument is not syllogistic at all unless it conforms to this structure. Since the number of propositions and terms is three, it is quite obvious that every term occurs twice. Consider an example for a syllogistic argument.

1st premise: All humans are stupid.
2nd premise: All sages are human.
Conclusion: Therefore all sages are stupid.

A term, which is common to the premises (human), is called *middle* (M). Predicate of the conclusion (stupid) is called *major* (P) and subject of the conclusion (sages) is called *minor* (S). While major has maximum extension, minor has minimum extension. The middle term is so called because its extension varies between the limits set by minor and major. The premise in which major occurs is called *major premise* and the premise in which minor occurs is called *minor premise*.

Though in this argument the first premise is major and the second is minor there is no rule which stipulates that this must be the order. Not only can minor premise be written first, but also the conclusion can as well be the first statement. The only restriction is that if an argument starts with premises, always 'therefore' or its synonym must precede the conclusion and if the conclusion is the starting point, then 'because' or its synonym must be immediately follow the conclusion. Aristotle argued that our inference proceeds from minor term to major term through middle term. Therefore in the absence of middle term, it is impossible to proceed from minor to major. Aristotle is also a pioneer who discovered predicate logic. He restricted syllogism to subject-predicate logic and, naturally he did not give credence to other forms of proposition like relational prepositions. Most of what Aristotle said on syllogism holds good only when we consider predicate logic (see below, block 4, unit 4).

## 3.3 AXIOMS OF SYLLOGISM

There are two types of axioms: axioms of quantity and axioms of quality. Rules under these axioms are merely stated because there is no proof to these rules.

A. Axioms of Quantity:

$A_1$: The middle must be distributed at least once in the premise.

$A_2$: A term, which is undistributed in the premise, must remain undistributed in the conclusion. A term, which is distributed in the conclusion, should compulsorily be distributed in the premise.

B. Axioms of quality:

$B_1$: Two negative premises do not yield any conclusion.

$B_2$: Affirmative premises yield only affirmative conclusion.

$B_3$: Negative premise (there can be only one negative premise) yields only negative conclusion.

Three corollaries follow from these rules. They are as follows: -

1. The number of terms distributed in the conclusion must be one less than the number of terms distributed in the premises. It is very easy to explain this corollary. The number of terms in the conclusion itself is one less than the number of terms in the premises and M which is compulsorily distributed in the premises is not a part of the conclusion.
2. Two particular premises do not yield any conclusion. Only one particular premise is permissible.
3. Particular premise yield only particular conclusion. [The reader is advised to prove these corollaries with the help of Axioms of quality and quantity.]

## 3.4 FIGURES AND MOODS

In the conclusion, S and P have fixed positions but this is not the case with M. There are four ways in which M can occupy two places. These four ways are called four figures, i.e., the position of M determines the figure of argument. These figures are as follows: -

| | **I** | **II** | **III** | **IV** |
|---|---|---|---|---|
| **Major Premise:** | M-P | P-M | M-P | P-M |
| **Minor Premise:** | S-M | S-M | M-S | M-S |
| **Conclusion:** | S-P | S-P | S-P | S-P |

From this scheme it is clear that neither P nor S determines the figure of syllogism. History has recorded that Aristotle accepted only first three figures. The origin of the fourth figure is disputed. While Quine said that Theophrastus, a student of Aristotle, invented the fourth figure, Stebbing said that it was Gallen who invented the fourth figure. This dispute is not very significant. But what Aristotle says on the first figure is significant.

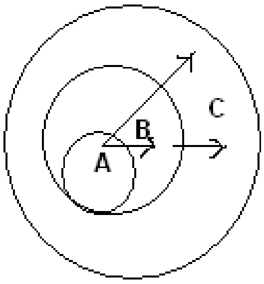
Aristotle regarded the first figure as most 'scientific'. It is likely that by 'scientific' he meant 'satisfactory'. One of the reasons, which Aristotle has adduced, is that both mathematics and physical sciences establish laws in the form of the first figure. Second reason is that reasoned conclusion or reasoned fact is generally found in the first figure. Aristotle believed that only universal affirmative conclusion can provide complete knowledge and universal affirmative conclusion is possible only in the first figure. Aristotle quotes the fundamental principle of syllogism. 'One kind of syllogism serves to prove that A inheres in C by showing that A inheres in B and B in C'. This principle can be expressed in this form:

Minor: A inheres in B

Major: B inheres in C

Conclusion: ∴ A inheres in C

Evidently, this argument satisfies transitive relation. This is made clear with the help of this diagram:




Let us mention four examples, which correspond to four figures.

## FIGURE I

| | M P | |
|---|---|---|
| Major Premise: | All Artists are Poets. | AAP |
| | S M | |
| Minor Premise: | All Musicians are Artists. | MAA |
| Conclusion: | ∴All Musicians are Poets. | MAP |
| | S P | |

## FIGURE II

| | P M | |
|---|---|---|
| Major Premise: | All saints are pious. | SAP |
| | S M | |
| Minor Premise: | No criminals are pious. | CEP |
| Conclusion: | ∴ No criminals are saints. | CES |
| | S P | |

## FIGURE III

| | M P | |
|---|---|---|
| Major Premise: | All great works are worthy of study. | GAW |
| | M S | |
| Minor Premise: | All great works are epics. | GAE |
| Conclusion: | ∴ Some epics are worthy of study. | EIW |
| | S P | |

## FIGURE IV

| | P M | |
|---|---|---|
| Major Premise: | No soldiers are traitors. | SET |
| | M S | |
| Minor Premise: | All traitors are sinners. | TAS |

Conclusion: ∴ Some sinners are not soldiers. SOS

S P

We have to consider figures in conjunction with moods. Mood is determined by quality and quantity propositions, which constitute syllogism. Since there are four kinds of categorical proposition and there are three places where they can be arranged in any manner, there are sixty-four different combinations in any given figure. Since there are four figures, in all, two hundred and fifty six ways of arranging categorical propositions are possible. These are exactly what we mean by moods. However, out of two hundred and fifty-six, two hundred and forty-five moods can be shown to be invalid by applying the rules and corollaries. So we have only eleven moods. There is no figure in which all eleven moods are valid. In any given figure only six moods are valid. They are as follows:

| | | | | | | |
|---|---|---|---|---|---|---|
| I. | AAA, | AAI | AII | EAE | EAO | EIO |
| II. | AEE | AEO | EAE | EAO | EIO | AOO |
| III. | AAI | AII | IAI | EAO | EIO | OAO |
| IV. | AAI | IAI | AEE | AEO | EAO | EIO |

In all these cases, first letter stands for major premise, second for minor and third for conclusion. Moods are boxed in two ways. Moods within thick boxes are called strengthened moods, and moods within thin boxes are called weakened moods. It is important to know the difference between these two. *When two universal premises can yield only particular conclusion, then such moods are called strengthened moods. On the other hand, if we deduce particular conclusion from two universal premises, when it is logically possible to deduce a universal conclusion, then such moods are called weakened moods*. When we recall that from universal premises alone particular conclusion cannot be drawn, both strengthened and weakened moods become invalid. Thus, the number of valid moods reduces to fifteen. In this scheme, we notice that EIO is valid in all the figures.

Though EIO is valid in all figures, it is one mood in one figure and some other in another figure. Likewise, AEE is valid in the second and the fourth figures. But it is one mood in the second figure and different mood in the fourth figure. In the thirteenth century, one logician by name Pope John XXI, invented a technique to reduce arguments from other figures to the first figure. This technique is known as mnemonic verses. Accordingly, each mood, excluding weakened moods, was given a special name:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| I. | Fig: | AAA | BARBARA | III. | Fig: | AAI | DARAPTI |
| | | EAE | CELARENT | | | IAI | DISAMIS |
| | | AII | DARII | | | AII | DATISI |
| | | EIO | FERIO | | | EAO | FELAPTON |
| | | | | | | OAO | BOCARDO |
| | | | | | | EIO | FERISON |
| II. | Fig: | EAE | CESARE | IV. | Fig: | AAI | BRAMANTIP |
| | | AEE | CAMESTRES | | | AEE | CAMENES |
| | | EIO | FESTINO | | | IAI | DIMARIS |
| | | AOO | BAROCO | | | EAO | FESAPO |
| | | | | | | EIO | FRESISON |

Syllogism can be tested using rules and corollaries. These are also known as general rules. There is one more method of testing syllogism. Every figure is determined by special rules. These are called special rules because they apply only to particular figure. These special rules also depend directly upon the axioms of quantity and quality. Therefore special rules can be proved. While doing so we shall follow the method of *reductio ad absurdum* because, it is a simple method.

I. **Special rules of the first figure**:

M – P
S – M
S – P

1. Minor must be affirmative:

   Proof :

   1. Let minor be negative.
   2. Conclusion must be negative. (From $B_3$ and 1)
   3. Conclusion distributes P. (From 2)
   4. Major should distribute P. (From $A_2$ and 3)
   5. Major must be negative. (From $A_2$ and 4)
   6. Negative minor implies negative major.
   7. Two premises cannot be negative ($B_1$)
   8. ∴Minor must be affirmative. q.e.d.

2. Major must be universal:

   Proof:

   1. Let Major be particular.
   2. Major undistributes M. (From 1)
   3. Minor should distribute M. (From $A_1$ and 1)
   4. Minor should be affirmative. (First special rule)
   5. ∴Minor has to undistributed M.
   6. ∴Major should distribute M. (From $A_1$)

7. ∴Major must be universal. q.e.d.

Using these two special rules, valid moods can be distinguished from invalid moods.

II. **Special rules of the Second figure**:

P – M
S – M
S – P

1. Only one premise must be negative:

   Proof:

   1. Let both premises be affirmative.
   2. M is undistributed in affirmative statements.
   3. (1) and (2) together contradict $A_1$.
   4. ∴One premise must be negative. q.e.d.

2. Major should be universal:

   Proof:

   1. Let Major be particular.
   2. Major undistributes P. (from 1)
   3. Conclusion must be universal. (From $B_3$ and first special rule).
   4. ∴Conclusion distributes P.
   5. (2) and (4) together contradict $A_2$.
   6. ∴ Major should distribute P.
   7. ∴ Major must be universal.

III. **Special rules of the Third figure**:

M – P
M – S
S – P

1. Minor must be affirmative.
2. Conclusion must be particular.

(The reader is advised to try to prove these two rules).

IV. **Special rules of the Fourth figure**:

P – M
M – S
S – P

1. If Major is affirmative, then minor must be universal.

   Proof:

   1. Let minor be particular when major is affirmative.

2. Major undistributes M.
3. Minor also undistributes M. (From 1)
4. (2) and (3) together contradict $A_1$.
5. ∴Minor should distribute M.
6. ∴Minor must be universal.

2. If any premise is negative, major must be universal.
Proof:

1. Let major be particular, when one premise is negative.
2. Negative premise yields negative conclusion. ($B_3$)
3. Negative conclusion distributes P.
4. Major should distribute P. (From 3 and $A_2$)
5. Major must be universal.
6. (1) and (5) contradict one another.
7 ∴Major must be universal. q.e.d.

3. If minor is affirmative, then, conclusion must be particular.
Proof:

1. Let conclusion be universal with affirmative minor.
2. Universal conclusion distributes S.
3. Minor should distribute S. (From $A_2$ and 2)
4. Affirmative minor undistributed S.
5. (3) and (4) contradict one another.
6. ∴Conclusion should undistribute S.
7. ∴Conclusion must be particular.

## 3.5 FALLACIES of Categorical Syllogism

There are three important fallacies associated with categorical syllogism. They are fallacies of undistributed middle, illicit major and illicit minor. One example for each fallacy with explanation will suffice.

| | P | M | |
|---|---|---|---|
| Major Premise: | All inscriptions are contents of historical study. | | IAC |
| | S | M | |
| Minor Premise: | All ancient coins are contents of historical study. | | AAC |

Conclusion: ∴ All ancient coins are inscriptions. AAI

Ans: This argument is in the second figure. According to one special rule of the second figure, only one premise must be negative. Since this rule is violated M is undistributed in both the premises.

∴ The argument commits the fallacy of undistributed middle.

While mentioning the rule violated we can also say that according to one axiom of quantity, M should be distributed at least once. When this rule is violated this fallacy is committed.

| | | |
|---|---|---|
| | M P | |
| Major Premise: | All sailors are strong. | SAS |
| | M S | |
| Minor Premise: | All sailors are men. | SAM |
| | S P | |
| Conclusion: | ∴All men are Strong. | MAS |

Ans: This argument is in the third figure. According to one special rule of the third figure, the conclusion must be particular. Since this rule is violated, the argument commits the fallacy of illicit minor. [The reader is advised to identify the second type of explanation.]

| | | |
|---|---|---|
| | P M | |
| Major Premise: | Some rich people are merchants. | RIM |
| | M S | |
| Minor Premise: | No merchants are educated. | MEE |
| Conclusion: | ∴Some educated persons are not rich. | EOR |

Ans: This argument is in the fourth figure. According to one special rule of the fourth figure, when a premise is negative major must be universal. This rule is violated by the argument and it commits the fallacy of illicit major. [The reader is advised to identify the second type of explanation.]

In any deductive argument certain elements are constant. In syllogism, for example, quality and quantity and position of terms determine the structure of the argument. Keeping the structure constant if any term is replaced by any other term, the end result remains the same. Therefore the student can construct as many examples as he or she wants. The method of identifying the fallacy remains the same, if the structure remains the same.

## 3.6 REDUCTION OF ARGUMENTS

Reducing arguments from other figures to the first figure is one of the techniques developed by Aristotle to test the validity of arguments. It is because Aristotle held that the first figure is the perfect one; all others are imperfect. After reduction, if the argument is valid in the first figure, then it means that the original argument in any other figure is valid. This technique is quite mechanical. So, we are only required to know what exactly is the method involved. We will learn this only by practice.

II Figure

| II Figure CESARE | | I Figure CELARENT |
|---|---|---|
| P E M | → Conversion→ | M E P |
| S A M | | S A M |
| S E P | | S E P |

| | | |
|---|---|---|
| No politicians are poets. | → Conversion → | No poets are politicians. |
| All girls are poets. | | All girls are poets. |
| ∴No girls are politicians. | | ∴No girls are politicians. |

In CESARE 'S' after 'E' indicates simple conversion. It shows that 'E' (major premise) must undergo simple conversion.

II Figure

| II Figure CAMESTRES | I Figure CELARENT |
|---|---|
| P A M | M E S |
| S E M → conversion M E S | P A M |
| ∴S E P | ∴P E S conversion→S E P |

'S' and 'T' after 'E' shows that 'E' (minor premise) should undergo simple conversion and both premises be transposed. 'S' after second 'E' shows that this 'E' (conclusion) should undergo simple conversion. [The student is advised to construct argument for this and subsequent reductions.]

II Figure

II Figure I Figure

$\overline{M}$ $\overline{M}$

$\overline{M}$ $\overline{M}$

V

$\overline{P}$

$\overline{P}$

$\overline{P}$

$\overline{P}$

$\overline{P}$

V o

c n

o

P E M → Conversion→ M E P

M A S → Conversion→ S I M

SOP SOP

As usual 'S' stands for simple conversion of 'E' (Major Premise) and 'P' stands for conversion per accedens of 'A' (Minor premise). This process is similar to first and third moods of III figure.

| FRESISON | | FERIO |
|---|---|---|
| P E M | → Conversion→ | M E P |
| M I S | → Conversion→ | S I M |
| S O P | | S O P |

A close observation of the above reductions reveals that they are to be performed according to certain parameters. The moods in the first figure are Barbara, Celarent, Darii and Ferio. Their initial consonants are arbitrarily found. For other figures, the initial consonants indicate to which of the first, the figure is to be reduced. Accordingly, Fesapo in the 4th figure is to be reduced to Ferio. Other consonants occurring in second, third and fourth figures' mnemonics indicate the operation that must be performed on the proposition indicated by the preceding vowel in order to reduce the syllogism to a first-figure syllogism. Certain 'keys' are the following. 's' indicates simple conversion; 'p' indicates conversion per accidens (by limitation); 'm' indicates the interchanging of the premises; 'k' indicates obversion; 'c' refers to the process that the syllogism is to be reduced indirectly. Meaningless letters in mnemonic terms are 'r', 't', 'l', 'n', and noninitial 'b' and 'd'.

From reduction technique one point becomes clear. Originally, there were twenty-four valid moods. Later weakened and strengthened moods were eliminated on the ground that particular proposition (existential quantifier) cannot be deduced from universal propositions (universal quantifier) alone, and the number was reduced to fifteen. Now after reduction to first figure the number came down to four. Strawson argues that reduction technique is superior to axiomatic technique to which he referred in the beginning of his work 'Introduction to Logical Theory'. He regards the moods as inference-patterns. He argues that the path of reduction should be an inverted pyramid. At one particular point of time Strawson maintains that in addition to equivalence relation, we require opposition relation also to effect reduction.

## 3.7 ANTILOGISM OR INCONSISTENT TRIAD

This technique was developed by one lady by name, Christine Ladd-Franklin (1847-1930). This technique applies only to fifteen moods. The method is very simple. Consider Venn's results for all propositions. Replace the conclusion by its contradiction. This arrangement constitutes antilogism. If the corresponding argument should be valid, then antilogism should conform to certain structure. It must possess two equations and one inequation. A term must be common to equations. It should be positive in one equation and negative in another. Remaining two terms appear in inequation. Consider one example for a valid argument.

| | **Venn's Results** | **Antilogism** |
|---|---|---|
| All Indians are Asians. | $I\bar{A} = Ø$ | $I\bar{A} = Ø$ |

I n
it n
ir s
a

N

1

$\overline{\mathrm{M}}$

2

$\overline{\mathrm{P}}$

3

$\overline{\mathrm{P}}$

4

$\overline{M}$

5

$\overline{M}$

6

$\overline{P}$

7

$\overline{M}$

$\overline{M}$

$\overline{P}$

8

$\overline{S}$

9

$\overline{P}$

1

$\overline{P}$

$\overline{S}$

$\overline{P}$

1

$\overline{P}$

1

$\overline{M}$

1

$\overline{S}$

1

$\overline{P}$

V

$\overline{P}$

$\overline{S}$

I a
w

$\overline{M}$

$\overline{M}$

A d
a

3

L d
f a
2 s.
S d
t st
b e
c a
u it
s e
p e
s it
w

$\bar{P}$

$\bar{M}$

$\bar{P}$

T

$\overline{SMP}$

$\bar{P}$

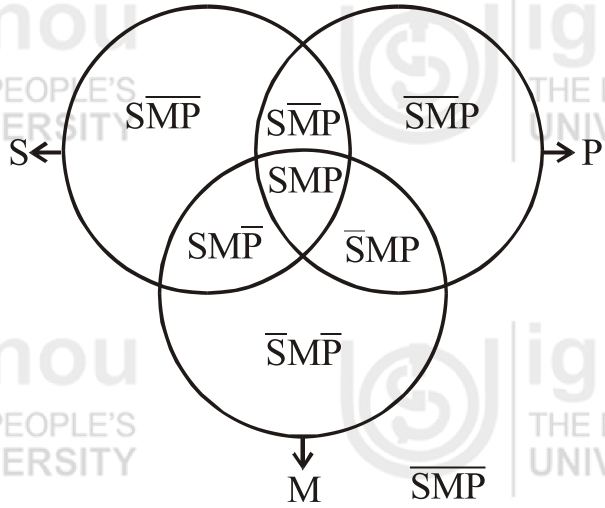
$\overline{MP}$

$\bar{S}$ $\bar{M}$

$\bar{S}$

$\bar{S}$ $\bar{P}$

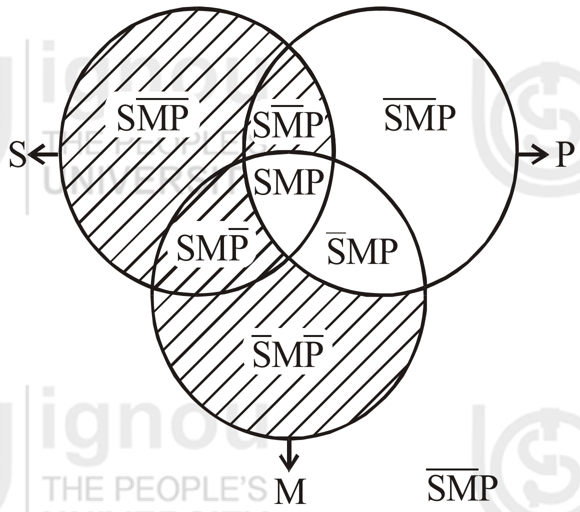
$\bar{M}$




W s
r s
a

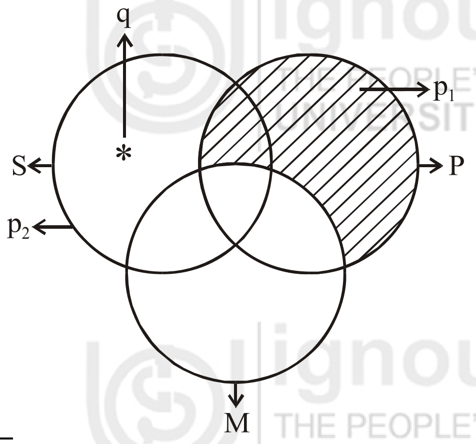
N d
n ll
s a
n

S $\bar{P}$ $\bar{P}$ $\bar{S}$ $\bar{P}$ e
a $\bar{M}$ e
p $\bar{M}$ $\bar{M}$ $\bar{P}$ e
i s,
v


S
P
M
$S\bar{M}\bar{P}$
$S\bar{M}P$
$\bar{S}\bar{M}P$
$SMP$
$SM\bar{P}$
$\bar{S}MP$
$\bar{S}M\bar{P}$
$\overline{SM}P$

q

$\overline{P}$




D

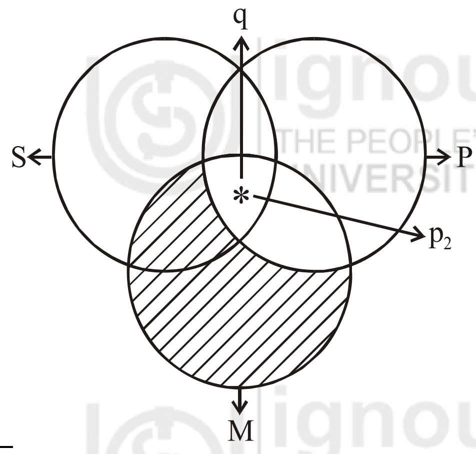
p

p

–

q

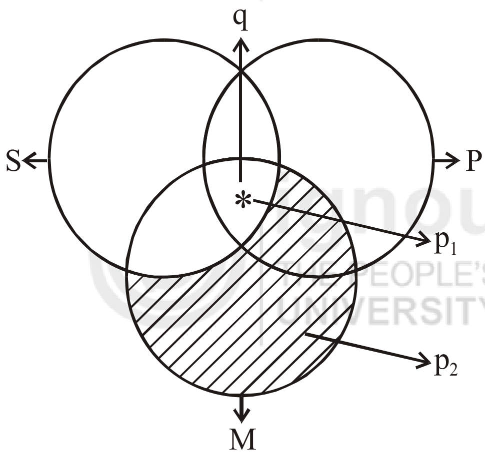
$\overline{P}$




D

p
p
–
q

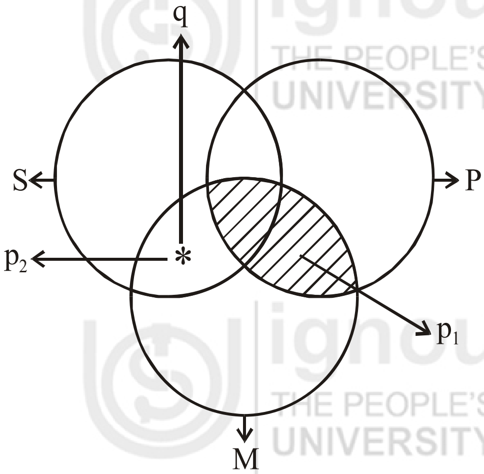
$\bar{S}$




F

p
p
–
q

$\bar{P}$



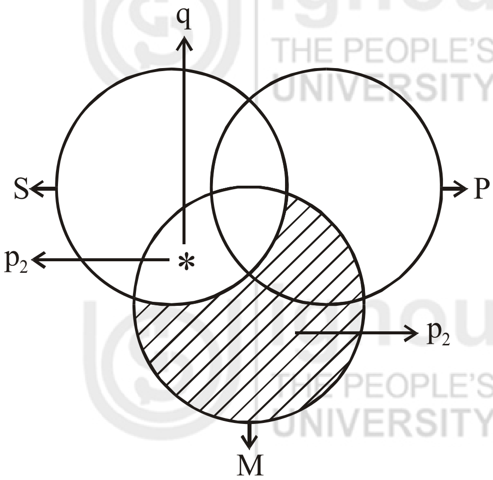
E

p
p
–

q

$\overline{P}$
$\overline{S}$

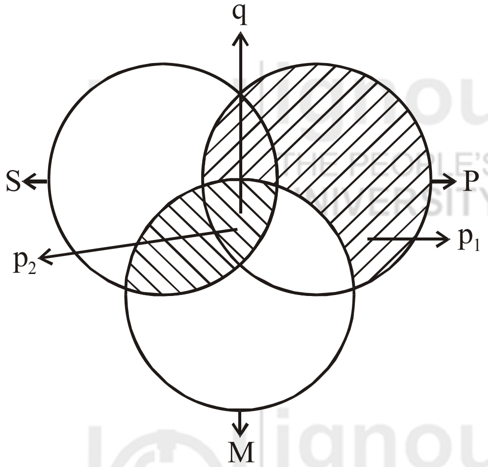
$\overline{P}$




C

p
p
–
q

$\overline{M}$



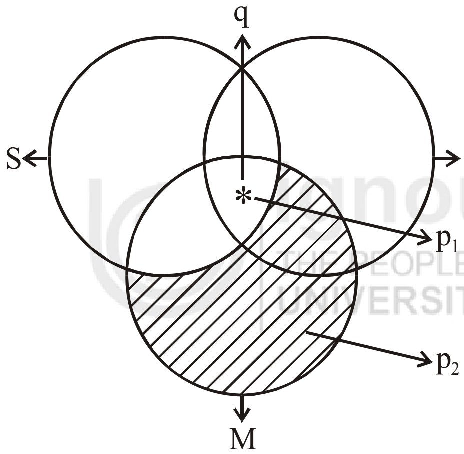

D

p

p

q

$\bar{S}$

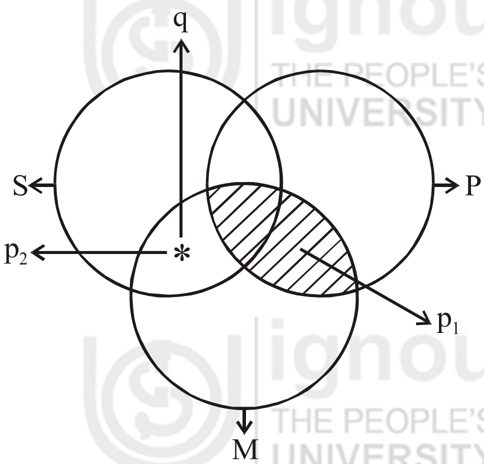



F

p

p

q $\overline{P}$



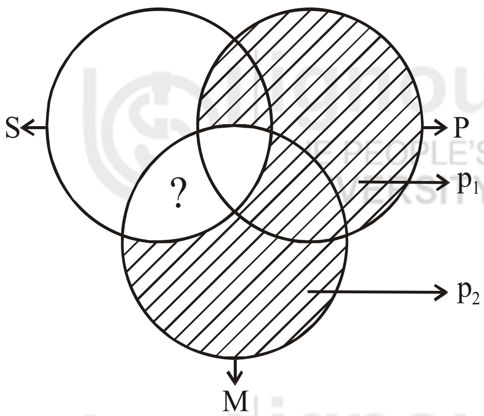

U

E

p $\overline{M}$

p $\overline{S}$

q



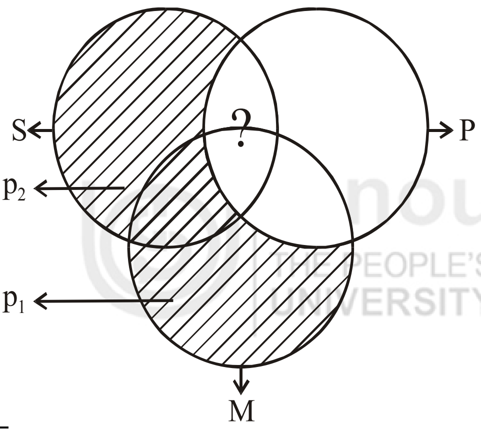

N $\overline{P}$ e

E

A

p $\overline{P}$
p $\overline{M}$

q




I $\overline{M}$ e
d
U

C

N

1

.

.
2

3 .
.

3

S n
d f

syllogism. There are five techniques to test the validity of arguments. Conditions of validity differ from traditional analysis to modern analysis.
There are three important fallacies in this category.

## 3.10 KEY WORDS

**Paradox:** A paradox is a statement or group of statements that leads to a contradiction or a situation which defies intuition or common experience.

## 3.11 FURTHER READINGS AND REFERENCES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## 3.12 ANSWERS TO CHECK YOUR PROGRESS

1. The rule which is common to conversion and syllogism is: 'term which is undistributed in the premise must remain undistributed in the conclusion'.
2. EIO is the only mood which is valid in all the figures.
3.IEO is invalid in all the figures.

## UNIT 4 TRUTH - FUNCTIONAL FORMS

**Contents**



## 4.0 OBJECTIVES

The aim of this unit is to introduce you to the concept of equivalence through two means; truth-table method and stroke and dagger function and contradiction through truth-table means. Though what you learn in this unit is much limited in terms of content, it forms the foundation of future learning. Hence this unit should prepare you to grasp the essence of the next block.

After you are thorough with this unit you should be in a position to:

- construct truth-tables for statements.
- identify propositions having different form but same content.
- reduce all verbal expression to non-verbal forms.
- discover that verbal form is more complex and not necessarily useful when compared with symbolic form, which is simpler and more useful in our logical enterprise.

## 4.1 INTRODUCTION

In Unit 2 we learnt that in our study of symbolic logic we replace propositions by variables. These variables may be called prepositional variables because they signify indifferently any statement. Therefore whenever a propositional variable is assigned any truth-value, then the same truth-value has to be assigned to any proposition signified by the respective variable. We also learnt that sentential connectives help us to obtain compound propositions. While statements are variables, various connectives like 'not', 'if…then', etc., which produce compound propositions, are logical constants. A study of symbolic logic starts with what is known as, 'calculus of propositions or propositional calculus'. There are different forms of truth-function, which constitute propositional calculus with which we have to familiarize. In other words, various relations between propositions require to be studied.

It is good to recapitulate what was discussed under compound statements. There are five kinds of compound propositions: implicative, conjunctive, disjunctive, negation and biconditional; each one defined by a definite form. An important aspect, which follows this discussion, is 'two kinds of relation which exist between these forms'. Contradiction and logical equivalence (equivalence in brief) are these forms with which we are concerned. The beginning of this study marks the beginning of the study of symbolic logic. Let us make a beginning with implication.

## 4.2 IMPLICATION AND ITS EQUIVALENT FORMS

Let p stands for 'there is increase in supply' and q stands for 'the prices will fall'. Then, as we know already, the statement, 'if there is increase in supply, then the price will fall' is an implication (material implication to be precise) in a standard form. Our task is to derive its various equivalent forms and contradiction. As usual, we shall construct truth-table and then go to verbal form:

*Table: 1*

| | | | | | Implication | Disjunction | Negation | |
|---|---|---|---|---|---|---|---|---|
| | **p** | **q** | **¬p** | **¬q** | **p=>q** | **¬p v q** | **¬ (p** | **Λ ¬q)** |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 2 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 3 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 0 |
| 4 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 |

Under negation there are two columns which reflect truth-values. It must be remembered that the last but one column stands for equivalence relation. Therefore care should be taken to write the truth-value of negation exactly under the negation sign.The advantage of truth-value method is obvious. The equivalence relation, which exists between implication and disjunction, is self-explanatory. However, relation with negation requires some clarification. There are two columns under negation, which reflect truth-values. Suppose that we ignore negation sign and corresponding truth-values and consider the last column then we are not considering negation but conjunction. The last column is the same as the following one:

| | p Λ ¬q |
|---|---|
| 1 | 0 |
| 2 | 1 |
| 3 | 0 |
| 4 | 0 |

However, the required form is not conjunction but negation. The truth-value of negation, of course, truth-functionally depends upon the truth-value of conjunction form. Therefore while selecting the column, which corresponds to negation form, we should exercise a little caution.

Now we shall consider the verbal form of logical equivalence. Suppose that the given proposition is as follows:

1) 'If there is increase in supply, then the prices will fall'. The components of this proposition and their symbols are as follows:

a). There is increase in supply. p

b). The prices will fall. q

The form of given proposition is as follows:

$$p \Rightarrow q \quad -- \quad (1)$$

If, instead of considering the form of any proposition, we symbolize propositions themselves, then we shall choose the first letter of the first term (in which case we ignore article, verb, etc.). In such a case we have to use upper-case letters. Then (1) is replaced by: I=>P. For some time let us use both the form of the proposition and symbols of given propositions.

| | Implication | | Disjunction | | Negation |
|---|---|---|---|---|---|
| a) | $p \Rightarrow q$ | ≡ | $\neg p \vee q$ | ≡ | $\neg (p \wedge \neg q)$ |
| b) | $I \Rightarrow P$ | ≡ | $\neg I \vee P$ | ≡ | $\neg (I \wedge \neg P)$ |

Now we have to consider the negation of (a) and (b) mentioned above.

a') "There is no increase in supply" or
"It is not the case that there is increase in supply": $\neg p / \neg I$

b') "The prices will not fall" or
"It is not the case that the prices will fall": $\neg q / \neg p$

Disjunction: It is not the case that there is increase in supply or the prices will fall:
$\neg p \vee q / \neg I \vee P$

Negation: It is not the case that both there is increase in supply and the prices do not fall:
$\neg (p \wedge \neg q) / \neg (I \wedge \neg P)$

When negation is expressed in words, it is very important to observe that after 'it is not the case that' the word 'both' should invariably be used. Otherwise a mistake will be made. In fact, the word 'both' stands for the verbal expression of parentheses.

Implication has one more equivalent form called contraposition. Its structure is as follows:

*Table: 2*

| **p** | **q** | **$\neg$ p** | **$\neg$ q** | **Implication** **p => q** | **Contraposition** **$\neg$ q=> $\neg$ p** |
|---|---|---|---|---|---|

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 1 | 1 |
| 2 | 1 | 0 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 0 | 1 | 1 |
| 4 | 0 | 0 | 1 | 1 | 1 | 1 |

Since sentential connective remains the same, the type proposition also remains the same. Hence its use is somewhat limited to the test of arguments.

## 4.3 DISJUNCTION AND ITS EQUIVALENT FORMS

If implication has equivalent disjunctive form, the converse also should hold good. The component proposition, 'there is increase in supply' and 'the prices will fall' are connected by the connective 'OR' and we obtain compound proposition as follows:
'There is increase in supply or the prices will fall'. Let us construct the truth table to be followed by verbal form.

*Table: 3*

| | | | | | **Disjunction** | **Implication** | **Negation** | |
|---|---|---|---|---|---|---|---|---|
| | **p** | **q** | **⌐ p** | **⌐ q** | **p v q** | **⌐ p => q** | **⌐ ( ⌐ p Λ ⌐q)** | |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 2 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 3 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 0 |
| 4 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 |

There is no difference in explanation for negation compared with implication. However, we shall repeat only truth-table form in order to eliminate any iota of doubt, if any. Accordingly, rewrite the truth-value of the last column.

| | **⌐ p Λ ⌐q** |
|---|---|
| 1. | 0 |
| 2. | 0 |
| 3. | 0 |
| 4. | 1 |

Evidently, what we have here is only conjunction, but what we want is negation. Therefore the set of relevant truth-values belong to the last but one column, which is truth-functionally dependent upon those of the last column.

Let us switch over to verbal form and begin from disjunction.

2. There is increase in supply or the price will fall.

We shall rewrite the components and then append their negations.

a) There is increase in supply. p/I
b) The prices will fall. q/P

-a) There is no increase in supply or it is not the case that there is increase in supply.
⌐p/⌐I

-b) The prices will not fall. ⌐q/⌐P

Implication: If it is not the case that there is increase in supply, then the prices will fall.
⌐ p => q or ⌐I => P

Negation: It is not the case that both there is no increase in supply and the prices will not fall. ⌐ (⌐ p Λ ⌐ q) or ⌐ (⌐I Λ ⌐P)

As in the case of implication, in this case also:

| Disjunction | | Implication | | Negation |
|---|---|---|---|---|
| p v q | ≡ | ⌐ p => q | ≡ | ⌐ (⌐ p Λ ⌐ q) |

Unlike implication, disjunction allows simple transposition of disjunctions. ∴ p v q ≡ q v p. In this case also transposition has limited application in the test of arguments. The rule which governs such simple transposition is known as rule of commutation. Therefore when we construct disjunctive syllogism, we are free to choose any component.

The relation between (p v q) and ⌐ (⌐ p Λ ⌐ q) is explained by what is known as de Morgan's law. It says that equivalence of disjunction consists in the negation of the conjunction of the negation of components. It is very important to understand this law completely and clearly. Here negation and conjunction are algebraic functions. Conjunction is equivalent to multiplication. We know that in algebra parenthesis also is equivalent to multiplication. Therefore negation within parentheses goes and negation outside parentheses remains. It shows that it is inadmissible to cancel three negation signs. To put it symbolically, ⌐ (⌐ p Λ ⌐ q) ≠ (p Λ q). The method of testing this inequality is very simple.

**Table: 4**

| | **p** | **q** | **⌐ p** | **⌐ q** | **Negation ⌐(⌐ p Λ ⌐q)** | | **Conjunction p Λ q** |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| 2 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 4 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |

Since the truth-value of these expressions is not the same in all instances, they are not identical.

## 4.4 NEGATION AND ITS EQUIVALENT FORMS

The equivalent forms of negation take implication and disjunction forms when suitably translated. We have two components, ‘there is increase in supply’ and ‘the prices will fall’. When connected by ‘not’ we obtain,

3) It is not the case that both there is increase in supply and the prices will fall.

Let us rewrite the components and append their negations.

a) There is increase in supply. p/I
b) The prices will fall. q/P

⌐ a) There is no increase in supply or it is not the case that there is increase in supply.
⌐ p/⌐I

⌐ b) The prices will not fall or It is not the case that the prices will fall. ⌐q/⌐P

Now construct the truth-table for equivalent forms.

**Table: 5**

| | **P** | **q** | **⌐ p** | **⌐ q** | **Negation ⌐ (p Λ q)** | | **Implication p => ⌐ q** | **Disjunction ⌐ p v ⌐ q** |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 2 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| 3 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 |
| 4 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 |

The verbal forms of relations are as follows:

Implication: If there is increase in supply, then the prices will not fall.
p => ⌐q / I => ⌐ P

Disjunction: There is no increase in supply or the prices will not fall:
⌐ p v ⌐ q / ⌐I v ⌐P

For negation also we do not consider transposition of components because it does not have any special significance. If the equivalent negation form of disjunction is given by de Morgan’s law, then converse also naturally holds good. What is negated is negation of ‘Conjunction’. That is, {⌐ (p Λ q)} is negated. Therefore negation sign goes. Conjunction is replaced by disjunction and components are replaced by their negations. Hence, we get disjunction, which is equivalent to negation.

Before we pass on to check contradiction, it is good to challenge our own choice. Let us start with implication. How can we assert that only ⌐p v q is equivalent to p=>q? Why cannot we say that p v ⌐q is also equivalent? It is nearly impossible to give explanation in verbal form, as to how ⌐ p v q is equivalent to p => q, but not p v ⌐q. If we compare the truth-values of ⌐ p v q and p v ⌐q with p => q, then the solution becomes clear.

**Table: 6**

| | p | q | ¬p | ¬q | p=>q | ¬p v q | p v ¬q | ¬p v ¬q |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 2 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 |
| 3 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 4 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |

¬ p v ¬ q is added only to reinforce our position. An equivalent expression must be true in only those instances in which the original expression is true (and in all such instances) and it must be false in only those instances in which the original expression is false (and in all such instances). According to this criterion, only ¬ p v q is equivalent disjunctive form to the original implication. The students are advised to test all other cases, like disjunctive proposition, using truth-table method to conclude that other than those mentioned are not equivalent to the original expression. At this stage, it should become clear that use of verbal expressions to determine their equivalent forms renders the task an uphill task and sometimes practically impossible. It is left to the students to verify the last statement which he can do by considering fairly a complex statement.

Now construct the scheme of equivalent expressions with truth- table.

**Table: 7**

| | p | q | ¬p | ¬q | | Implication | Disjunction | Negation |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | Implication: p => q | ¬ q => ¬ p | ¬ p v q | ¬(p Λ ¬q) |
| 2 | 1 | 0 | 0 | 1 | Disjunction p v q | ¬ p => q | (q v p) | ¬p Λ ¬q |
| 3 | 0 | 1 | 1 | 0 | Negation ¬ (p Λ q) | p=> ¬q | ¬ p v ¬q | ¬ (q Λ p) |

It must be noted that while implication does not take equivalent converse form, disjunction and negation take.

## 4.5 CONJUNCTION AND BICONDITION

It is quite interesting to note that conjunction and bicondition do not have equivalent forms. Truth- table again comes to our rescue. It is sufficient if we consider any one-form, say, implication. If one equivalent form is absent, it is imperative that other forms are also absent.

**Table: 8**

| | p | q | ¬p | ¬q | p Λ q | p=>q | ¬p=>q | p=>¬q | ¬p=>¬q |
|---|---|---|---|---|---|---|---|---|---|
| **1** | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **2** | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 |
| **3** | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| **4** | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 |

Except that truth-values of conjunction do not tally with any possible arrangement in implication form, no other explanation is conceivable for the absence of equivalent forms to conjunction [The students are advised to test other forms to convince themselves].

Biconditional proposition also does not have any equivalent form. The reason is very simple. Biconditional is, in reality, conjunction only and both the conjuncts are implicative. First we shall know why it is regarded as conjunction.

**Table: 9**

| | | | | | **1** | **2** | **3** | **4** |
|---|---|---|---|---|---|---|---|---|
| | **p** | **q** | **⌐p** | **⌐q** | **p<=>q** | **(p=>q)** | **Λ** | **(q=>p)** |
| **1** | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| **2** | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| **3** | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| **4** | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |

The method of computing is as follows; first, we shall compute the truth-values of implication (p => q) and then we will compute the truth-values of q => p. These two sets of truth-values together determine the truth-value of conjunction. When we compare columns 1 and 3, we will come to know that these two expressions have identical truth-values in all instances. It shows that bicondition is also a conjunctive proposition where the conjuncts themselves are compound propositions. Therefore what applies to conjunction naturally, applies to bicondition also.

## 4.6 FORM OF CONTRADICTION

When arguments are to be tested, quite frequently, we look for contradiction. Therefore it is necessary that we should know the contradiction of compound propositions so that with ease we can detect contradiction in arguments. The rule of contradiction is as follows: Whenever p is true its contradiction is false and whenever p is false its contradiction is true. That is to say contradiction and negation are same. The truth-table for contradiction is as follows.

**Table: 10**

| | | | | | **Implication** | **Contradiction** |
|---|---|---|---|---|---|---|
| | **p** | **q** | **⌐p** | **⌐q** | **p=>q** | **p Λ ⌐q** |
| 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| 2 | 1 | 0 | 0 | 1 | 0 | 1 |
| 3 | 0 | 1 | 1 | 0 | 1 | 0 |
| 4 | 0 | 0 | 1 | 1 | 1 | 0 |

It is not difficult to express or understand contradiction in verbal form. We shall consider the components and their negation mentioned earlier.

Implication: If there is increase in supply, then the prices will fall.

| | | |
|---|---|---|
| a) | There is increase in supply: | p/I |
| b) | The prices will fall: | q/P |
| a') | There is no increase in supply: | ⌐ p /⌐ I |
| b') | The prices will not fall | ⌐ q /⌐ P |

Contradiction: There is increase in supply and the prices will not fall:

p Λ ⌐q or I Λ ⌐P

As we challenged earlier conclusion, we shall again challenge this conclusion also. How can we say that p Λ ⌐q is the only contradiction? How do we know that this is the only form of contradiction permissible? Contradiction, in this case, does not have equivalent relation because p Λ ⌐q is a conjunction and conjunction does not have equivalent forms. As a rule, for any given proposition there is only one form of contradiction. We shall consider one disjunction form:

**Table: 11**

| | | | | | **Implication** | **Disjunction** |
|---|---|---|---|---|---|---|
| | **p** | **q** | **⌐p** | **⌐q** | **p=>q** | **p v ⌐q** |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 |
| 2 | 1 | 0 | 0 | 1 | 0 | 1 |
| 3 | 0 | 1 | 1 | 0 | 1 | 0 |
| 4 | 0 | 0 | 1 | 1 | 1 | 1 |

In order to test the conclusion, we effected only one change; we replaced conjunction by disjunction. In first and fourth instances, we notice that the truth-value remained the same whereas it should have been different. Therefore p v ⌐q is not a contradiction of implication.

Contradiction of disjunction is, again, determined in accordance with de Morgan's law; replace disjunction by conjunction and disjuncts by their negations. Therefore the removal of negation prefixed to equivalent form of disjunction results in contradiction. The truth table is as follows:

**Table: 12**

| | | | | | **Disjunction** | **Contradiction** |
|---|---|---|---|---|---|---|
| | **p** | **q** | **⌐p** | **⌐q** | **p v q** | **⌐p Λ ⌐q** |
| 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| 2 | 1 | 0 | 0 | 1 | 1 | 0 |
| 3 | 0 | 1 | 1 | 0 | 1 | 0 |
| 4 | 0 | 0 | 1 | 1 | 0 | 1 |

The verbal form is as follows:

Disjunction: There is increase in supply or the prices will fall: p v q / I v P

Contradiction: There is no increase in supply and the prices will not fall:

⌐p Λ ⌐q / ⌐I Λ ⌐P

[In this case also contradiction does not have equivalent forms. If the student wishes to test other alternatives, he or she can follow the method suggested earlier.]
Contradiction of conjunction also is determined in accordance with de Morgan's law; replace conjunction by disjunction and the conjuncts by their contradictions.

**Table: 13**

| | **p** | **q** | **⌐p** | **⌐q** | **Conjunction** p Λ q | **Contradiction** ⌐p v ⌐q |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| 2 | 1 | 0 | 0 | 1 | 0 | 1 |
| 3 | 0 | 1 | 1 | 0 | 0 | 1 |
| 4 | 0 | 0 | 1 | 1 | 0 | 1 |

The verbal form is as follows:
Conjunction: There is increase in supply and the prices will fall: p Λ q / I Λ P
Contradiction: There is no increase in supply or the prices will not fall:
⌐ p v ⌐ q / ⌐I v ⌐P
Since contradiction is in disjunctive form it has equivalent implicative form. I => ⌐P, Ooviously, is its equivalent form.

The contradiction of biconditional proposition is indirectly found and it is in accordance with de Morgan's law since its conjunctive feature is only concealed (i.e., the biconditional is a conjunction of two conditionals, as we see under 2,3,4 in the table) . Let us start with truth-table. [The verbal form is left out so that the student can attend the same.]

**Table: 14**

| | | | | | **A** | | **B** |
|---|---|---|---|---|---|---|---|
| | | | | | | | Contradiction |
| | | | | | 1 | 2 3 4 | 5 6 7 |
| | P | q | ⌐p | ⌐q | p<=>q | (p=>q)Λ(q=>p) | (pΛ⌐q) v (qΛ⌐p) |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 1 1 | 0 0 0 |
| 2 | 1 | 0 | 0 | 1 | 0 | 0 0 1 | 1 1 0 |
| 3 | 0 | 1 | 1 | 0 | 0 | 1 0 0 | 0 1 1 |
| 4 | 0 | 0 | 1 | 1 | 1 | 1 1 1 | 0 0 0 |

Compare columns 3 and 6. It becomes clear that A and B are contradictories.
For the sake of clarity let us consider contradiction in more than one step.

Step: 1 Contradiction components

| | Given expression | Contradiction |
|---|---|---|
| a) | p=>q | p Λ ⌐ q |
| b) | q=>p | q Λ ⌐ p |

Replace given expression by their contradictions, we obtain: (pΛ⌐q)Λ(qΛ⌐ p)

Step: 2 In Step1, we apply one aspect of de Morgan's law, i.e., replacing conjunct by their negation. In step 2 we apply second aspect of de Morgan's law; i.e., replace conjunction by disjunction. We get: (pΛ⌐q) V (qΛ⌐p)

We are only required to compare columns 3 and 6 to assure ourselves that the chosen and tested form is the contradiction of the original expression.

We shall tabulate the results and at this stage we can omit the basic columns, i.e., truth-values of p, q, ⌐ p & ⌐q since we are familiar with the process involved.

| | | | Contradiction Form |
|---|---|---|---|
| a) | Implication | (p=>q) | p Λ ⌐q |
| b) | Disjunction | (p v q) | ⌐ p Λ ⌐q |
| c) | Conjunction | (p Λ q) | ⌐ p v ⌐q |
| d) | Bicondition | (p<=>q) | (p Λ ⌐q) Λ (q Λ ⌐p) |

It may be noted that when we compute equivalent forms we can do away with implication. We are at liberty to retain disjunction or conjunction. Only negation is constant. Since we can derive from negation and disjunction all other sentential connectives, these two one are called primitive connectives. (However, bicondition is an exception). Such a process results in a sort of simplification since the number of connectives we require comes down as a result of this process. In order to further reduce the number of connectives, a different technique was introduced. This is known as stroke and dagger operation.

## 4.7 THE STROKE FUNCTION (│)

Though the stroke function was introduced by C.S. Peirce, it is better known as the Sheffer-function after H.M. Sheffer, a mathematician. This function has negative force. The stroke function also is called stroke operator. This is also a connective because its use determines the truth-value of compound proposition, given the truth-value of its components.The definition of this function can be attempted in this fashion:

"When a stroke connects any two statements, then it has to be construed that at least one of them is false, if the function itself must be true." Suppose that p and q are statements forms. Then p│q means that either p is false or q is false when p│q is true. This definition does not rule out the possibility of both p and q being false. It can be depicted in the following manner:

**Table: 15**

| | **p** | **q** | **p │ q** |
|---|---|---|---|
| 1 | 1 | 1 | 0 |
| 2 | 1 | 0 | 1 |
| 3 | 0 | 1 | 1 |
| 4 | 0 | 0 | 1 |

Accordingly, compound propositions can be expressed in stroke form in the following manner:

1). Negation:

| Truth-table method | | Stroke Method |
|---|---|---|
| p | ¬p | p│p |
| 1 | 0 | 0 |
| 0 | 1 | 1 |

2). Conjunction

**Table: 16**

Truth-table method Stroke method

| | | | | | 1 | 2 3 4 |
|---|---|---|---|---|---|---|
| | **p** | **q** | **¬p** | **¬q** | **p∧q** | **(p│q) │ (p│q)** |
| 1 | 1 | 1 | 0 | 0 | 1 | 0 1 0 |
| 2 | 1 | 0 | 0 | 1 | 0 | 1 0 1 |
| 3 | 0 | 1 | 1 | 0 | 0 | 1 0 1 |
| 4 | 0 | 1 | 1 | 1 | 0 | 1 0 1 |

This process needs some explanation and explanation is in terms of truth-value. Consider p│q and apply the definition of stroke function. p│q is false only when both p and q are true, i.e. in the first instance only. In all other instances, from Table 14, we understand that at least one of them is false. So the stroke function is true. Now consider columns 2 and 4. Only in the first instance '0' appears in these two columns and nowhere else '0' appears in columns 2 and 4. Therefore in accordance with the definition of stroke function column 3 takes the value 1 only in the first instance. When we compare column (1) and (3) we learn that there is agreement in terms of the truth-value in all the instances. Therefore (p Λ q) ≡ (p│q) | (p│q), i.e., they are logically equivalent.

**Table 17**

3). Disjunction:

Truth-table method Stroke method

| | | | | | 1 | 2 3 4 |
|---|---|---|---|---|---|---|
| | p | q | ¬p | ¬q | p v q | (p│p) │ (q│q) |
| 1 | 1 | 1 | 0 | 0 | 1 | 0 1 0 |
| 2 | 1 | 0 | 0 | 1 | 1 | 0 1 1 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 3 | 0 | 1 | 1 | 0 | 1 | 1 1 0 |
| 4 | 0 | 0 | 1 | 1 | 0 | 1 0 1 |

Considering the fact that stroke function is somewhat subtle, explanation is desirable. Apply the definition of stroke function to p│p and q│q. p│p is true only in 3rd and 4th instances where p is false. According to the definition of stroke functions, stroke function is true only when at least one component is false. p│p is false 1st and 2nd instances when p is true. Similarly, q│q is true in 2nd and 4th instances when q is false. Now apply stroke function to column 3. It takes the value 1in the first three instances since 0 appears either in column 2 or column 4 in these instances. It can take the value '0' only in the fourth instance since only in this instance the columns 2 and 4 take the value1. When we compare columns (1) and (3) we learn that there is agreement in terms of the truth-value in all the instances. Therefore, (p v q) ≡ (p│p) | (q│q), i.e., they are logically equivalent.

3). Implication:

**Table 18**

Truth-table method Stroke method

| | | | | | 1 | 2 3 4 |
|---|---|---|---|---|---|---|
| | p | q | ⌐ p | ⌐q | p => q | p │ (q│q) |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 1 0 |
| 2 | 1 | 0 | 0 | 1 | 0 | 1 0 1 |
| 3 | 0 | 1 | 1 | 0 | 1 | 0 1 0 |
| 4 | 0 | 0 | 1 | 1 | 1 | 0 1 1 |

The truth-value, which appears in column 3 is truth-functionally dependent on truth-values, which appear in columns 2 and 4. Column 3 takes the value 1 in instances 1, 3 and 4.Since in these instances '0' appears in one or the other column. Only in second instance column 3 takes the value 0 since columns 2 and 4 both take the value 1.This is in accordance with the definition of stroke function. Columns 1 and 3 agree in all the instances in terms of truth-value. Therefore, (p=>q) ≡ p│ (q│q), they are logically equivalent.

## 4.8 THE DAGGER FUNCTION (↓)

The dagger version can be regarded as stronger variation of the stroke function. When a compound proposition is expressed in terms of stroke function, the rule is that at least one of the components must be false if the stroke function must be true, though the possibility of both being false to make stroke function true is allowed. However, in dagger function, both the components must be false to make it true. This statement is regarded as the definition of dagger function.

Suppose p and q are statements forms, then, p↓q is true if and only if both p and q are false. Otherwise, it is false. Accordingly, compound propositions can be expressed in dagger form in the following manner.

**Table: 19**

| | p | q | p ↓ q |
|---|---|---|---|
| 1 | 1 | 1 | 0 |
| 2 | 1 | 0 | 0 |
| 3 | 0 | 1 | 0 |
| 4 | 0 | 0 | 1 |

1) Negation:

Truth- table method

| p | ¬ p |
|---|---|
| 1 | 0 |
| 0 | 1 |

Dagger method

| p↓p |
|---|
| 0 |
| 1 |

It is clear that ¬ p ≡ p↓p. In this respect, the stroke and dagger functions concurr.

2.Conjunction:

**Table: 20**

| | | | | | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|
| | p | q | ¬p | ¬q | pΛq | (p↓p) | ↓ | (q↓q) |
| 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| 2 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 3 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| 4 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |

Columns (2) and (4) determine the truth-values in column (3) in accordance with the definition of the dagger function. Column (1) and (3) horizontally agree in terms of truth-values in all the instances. Therefore, (pΛq) ≡ (p↓p)↓(q↓q)

3). Disjunction:

**Table: 21**

| | | | | | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|
| | p | q | ¬ p | ¬q | p v q | (p↓q) | ↓ | (p↓q) |
| 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| 2 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 3 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |
| 4 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |

Explanation is left out so that the student can attempt the same.

4). Implication:

(p => q) ≡ {(p↓p)↓q}↓{(p↓p)↓q}

{

Explanation and truth-table are left out so that the student can attempt the same.

Biconditinal proposition can be expressed neither in stroke form nor in dagger form. So it does not have any form of equivalence. Negation of conjunction also does not have equivalence in these forms.

**Check Your Progress**

**Note:** a) Use the space provided for your answer.
b) Check your answers with those provided at the end of the unit.

Read carefully the questions and instructions before you answer.
Six statements are given. Answer as per directions.

1. Crow is black.
--------------------------------------------------------------------------------
2. Philosophy is the mother of all sciences.

--------------------------------------------------------------------------------
3. Walter Scott is the author of King Waverly.

--------------------------------------------------------------------------------
4. Japan is a small nation.

5. America is an ally of China.

--------------------------------------------------------------------------------
6. Brutus killed Caeser.

--------------------------------------------------------------------------------

Choose any two propositions. Make as many pairs as possible, like (1,2),(1,3),(2,6),(3,5), etc. Connect them using =>, Λ, v and <=>. The following assumptions have to be made.

| | $p_1$ | and | $p_2$ |
|---|---|---|---|
| 1 | True | | True |
| 2 | False | | False |
| 3 | True | | False |
| 4 | False | | True |

In this way assume the truth-values of components. After constructing compound proposition based on the truth-value you have assigned to components, determine their truth-value. Also, remember that for every assumption of truth-value there are four compound propositions. Two examples are given below:

(1). 1 – True 6 – True (2) 1 – False 6 – False
(3) 1 – True 6 – False (4) 1 – False 6 – True

For (1) you have to construct four compound propositions using four connectives given above. Likewise for (2) four compound propositions, for (3) four compound propositions etc. In all cases write equivalent forms, which include stroke and dagger functions and contradiction.

## 4.9 LET US SUM UP

Truth-function and variables are basic to propositional calculus. Symbolic logic begins with propositional calculus. Compound propositions are characterized by both variables and constants. Contradiction and equivalence are two important logical relations. While conjunction and bicondition do not have equivalent forms other compound propositions have equivalent forms. Equivalent forms eliminate all but two connectives, negation and disjunction, which are primitive connectives. Stroke and dagger operators reduce the number to one. Only bconditional remains unaffected.

## 4.10 KEY WORDS

Operator: In symbolic logic 'operator' means a tool with the help of which an action is performed. Here the act consists in determining the truth-value of a compound proposition.
Constant: A constant is a quantity that does not change, over time or otherwise. It has a fixed value.
Variable: A variable is a symbol for which there many suitable substitutions.

## 4.11 FURTHER READINGS AND REFERENCES


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## 4.12 ANSWERS TO CHECK YOUR PROGRESS

If all combinations are formed the number runs to about three-hundred. Since it is neither feasible nor required to work out all possibilities only a few are worked out for illustration purpose. In your own interest you should make as many combinations as you can.

1. Crow is black-1; ; philosophy is the mother of all sciences.-1
   Crow is black and philosophy is the mother of all sciences.-1
   Since both components are conjunction is true.
2. Crow is black-0; philosophy is the mother of all sciences.-1
   Crow is black and philosophy is the mother of all sciences.- 0
   Since one component is false, the conjunction is false.
3. Crow is black-0; philosophy is the mother of all sciences.-1
   If Crow is black, then philosophy is the mother of all sciences.-1

Since the antecedent is false and the consequent is true, the implication is true.
Crow is black-1; philosophy is the mother of all sciences.-0
If Crow is black, then philosophy is the mother of all sciences.-0
Since the antecedent is true and and the consequent is false, the implication is false.

4. Crow is black-1; philosophy is the mother of all sciences.-1
   Crow is black or philosophy is the mother of all sciences.-1
   The disjunction is true because both components are true.
5. Crow is black-0; philosophy is the mother of all sciences.-0
   Crow is black or philosophy is the mother of all sciences.-0
   The disjunction is false because both the components are false.
6. Crow is black-0; ; philosophy is the mother of all sciences.-0
   Crow is black if and only if philosophy is the mother of all sciences.-1
   The bicondition is true because both the components possess the same truth- value.
7. Crow is black-1; philosophy is the mother of all sciences.-0
   Crow is black if and only if philosophy is the mother of all sciences.-0
   The bicondition is false because the two components have different truth-values.
8. Crow is black-1; ; philosophy is the mother of all sciences.-1

It is not the case that both crow is black and philosophy is the mother of all sciences.-0
Negation of conjunction is false because both components are true.

9. Crow is black-1 and philosophy is the mother of all sciences.-0
   It is not the case that both crow is black and philosophy is the mother of all sciences.-1
   Since one of the components is false, the negation of conjunction is true.

**BLOCK-4 INTRODUCTION**

Symbolic logic is the study of argument forms. An *argument* is a unit of reasoning that attempts to prove that a certain idea is true by citing other ideas as evidence. The idea that the argument tries to prove true is called the "*ultimate conclusion*." Each argument has only one ultimate conclusion. Ideas that the argument uses as evidences for the ultimate conclusion are called "*premises*." Every argument has at least one premise and may have any number of them. Intermediate ideas on the way from the premises to the ultimate conclusion are called "*subconclusions*." An argument does not need to have any subconclusions, although most arguments do. If an argument has subconclusions, it can have any number of them. The present block, consisting of 4 units, deals with various forms of arguments in symbolic logic.

Unit 1 is on "Formal Proof of Validity: Rules of Inference." This unit makes explicit the technique of testing arguments. This is being achieved in two ways; exposing the limitations of traditional logic and simultaneously demonstrate the necessity of traditional logic. These twin objectives constitute the core of this unit.

Unit 2 highlights "Formal Proof of Validity: Rules of Replacement." This unit exposes the inadequacy of the rules of inference. It also demonstrates that logic is a growing science. If new techniques of testing arguments are invented, then logic stands on par with technological science where continuous inventions and improvements are the order of the day. This unit also serves to demonstrate a crucial factor that all arguments do not fall under one or two categories. Therefore, same set of rules cannot guarantee success.

Unit 3 explains the meaning of "Conditional Proof and Indirect Proof." This unit introduces a new list of techniques of testing the validity of arguments. There are as many kinds of arguments as there are techniques. The main purpose of this unit is to make you understand that there is no single technique which helps you to solve all kinds of problems. It is not sufficient if you know the art of testing validity only. Therefore, one has to know the art of testing invalidity also. To have a satisfactory knowledge of good argument you should also know what makes an argument bad.

Unit 4 discusses "Quantification." Broadly speaking, there are two types of arguments: arguments consisting of statements, which are truth-functionally compound and arguments, which are neither truth-functional nor compound. This unit deals with the latter kind of arguments. In this unit, a new set of rules is proposed to test the validity of arguments, which consist of general and singular propositions. This unit will help you to understand Aristotle's theory of syllogism against the background of symbolic logic.

The above given 4 units aim at providing each student with the ability to think rigorously, identify and dissect arguments, represent arguments in symbolic notation, and determine the validity of arguments using deductive proofs. A person makes an "argument" when he or she makes a claim and tries to back that claim up with some evidence. In other words, an argument consists of a claim and some reasons that are supposed to support the claim. Of course, you make and evaluate arguments all of the time, and probably with a good amount of skill. But in this course we step back and ask: what makes an argument good? What principles should we employ to discriminate between good and bad arguments?

# UNIT 1 FORMAL PROOF OF VALIDITY: RULES OF INFERENCE

**Contents**



## 1.0 OBJECTIVES

The main objective of this unit is:

- to make explicit the art of testing arguments. This is being achieved in two ways; the limitations of traditional logic are exposed and at the same time the necessity of traditional logic is being demonstrated.
- to compare verbal form of arguments and its symbolic form.
- to assess the relative merits and demerits of two forms if any.
- to translate symbolic representation to verbal form and verbal form to symbolic form.

## 1.1 INTRODUCTION

The primary function of logic is to classify arguments into good and bad. This can be done by testing the validity of arguments. As we know only limited types of arguments are covered by classical logic. Even those arguments which are within the range of modern logic are not alike in all respects. Some are simple enough so that the truth-table technique is adequate for the purpose of testing. Now, what is this truth-table technique of determining the validity of arguments? Let us take an argument form, for example:

If p, the q
p
Therefore, q

Its truth table can be constructed as follows: we need the initial columns of the statement or proposition variable p and q; then we need a column for the first premise here which is an implicative statement (second premise and conclusion are the initial columns themselves); since there are only two variable in this argument we need only four rows, as we have learned earlier. The truth-table of the above argument is as follows:

| | **p** | **q** | **p =>q** |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 2 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 |
| 4 | 0 | 0 | 1 |

Truth-table technique uses the principle that in a valid argument the conclusion is implied in the premises and so from true premises only true conclusions follow. In the above truth table, only in the first row the premises are true (see under p => q and p) and there under q we see the truth value as true. Hence we can say, in this truth-table no substitution instances (i.e., different rows) with true premises and false conclusion is seen and so it is a valid argument. This is a mechanical method; just construct a truth table for any given argument and see whether there are instances with true premises and false conclusions. Students can attempt the same for the following elementary arguments forms given below as rules of inference.

Generally, any argument, which consists of two or three simple but different propositions, is regarded as amenable to the truth-table method. But if the argument consists of more than three different propositions, then the truth-table method is of no avail. It is mainly because of its manoeuvrability, i.e., if there are three propositions, we need 8 rows in the truth-table; if four, then 16; if 5, the 32; if 6, the 64, and so on. In such circumstances we have to look for an alternative. Formal proof helps us here in quickly determining the validity of arguments. See the following example:

1. A => B
2. B => C
3. C => D
4. ¬D
5. A v E / ∴ E
6. A => C
7. A => D
8. ¬A
9. E

In this argument we have five propositions like A, B, C, D, E; if we construct truth-table for it, we need 32 rows. Now we can prove its validity by applying certain rules of inference in just four lines. This is exactly the advantage of formal proofs.

An argument, which is complex in this sense, is nothing but an aggregate of several simple (by simple, in this context, we mean short) arguments. Examples make this point clear.

1) p => q
p
————
∴ q

2) q => r
q
————
∴ r

3) p =>q
q => r
————
∴ p => r

In classical logic also we have 'complex' type of argument in the form of sorites. (We should remember that the terms complex, simple, etc. are relative). An example for sorites is given:

1 All Indians are Asians.
All Hindus are Indians.
All Kannadigas are Hindus. .
∴ All Kannadigas are Asians.

There are three premises and a conclusion. Hence, it is a polysyllogistic argument. As a matter of fact, a sorites consists of at least two syllogistic arguments and therefore, two conclusions. So it is more complex than an ordinary syllogism. This point becomes clear when we break sorites into constituent syllogisms.

2a).

All Indians are Asians.
All Hindus are Indians. ] → All Hindus are Asians.
All Kannadigas are Hindus. → All Kannadigas are Hindus.
∴ All Kannadigas are Asians

Fortunately or unfortunately, we hardly encounter such stereotype arguments. So there is need to sharpen and augment the tools of testing. At this critical juncture, it is very important to remember that no rule stipulated by classical logic can be ignored or violated. It is the foundation on which the superstructure, i.e., modern logic is built. For the sake of convenience, let us restrict ourselves only to symbols and go to verbal form when we take up exercise.

## 1.2 FORMAL PROOF OF VALIDITY: IT'S MEANING

In modern logic an argument is regarded as a sequence of statements. When proof is constructed to test the argument, the proof also takes the same form, which the argument takes. In this type of proof there is correspondence between the scheme of the given argument and the scheme of the proof. Every step, which is adduced while constructing proof, is the conclusion of the preceding statements, and in turn, becomes the premise for statements, which follow it (if not all, at least to some). Rules, which govern the process of deducing hidden conclusion, constitute what are known as 'Rules of Inference' in modern logic. Many of these rules have their origin in traditional logic.

There is a certain way of constructing proof in modern logic. More descriptive method, which consumes both space and time, has given way to much shorter and simpler method. Whatever conclusion can be drawn from any two given premises is written on left hand side (LHS) while the rule and the premises to which this particular rule applies to derive the conclusion used in further proof, are written on the right hand side (RHS). A rule of inference is applied to the whole line. This is an important point to note. As an economy measure, instead of premises, corresponding serial numbers are written. Thereby we save time. We must ensure that drawn conclusion, the respective premises and the rule applied are always juxtaposed. This procedure is the simplest and most economical in terms of time and effort to grasp the argument.

## 1.3 RULES OF INFERENCE

Our task, from now onwards, is very simple. Modern logic considers nine rules of inference. They are listed below.

**1) Modus Ponens (M.P.)**

p => q
p
∴ q

**2) Modus Tollens (M.T.)**

p => q
⌐ q
∴ ⌐ p

**3) Hypothetical Syllogism (H.S.)**

p => q
q => r
∴ p => r

**4) Disjunctive Syllogism (D.S.)**

p v q
⌐ p
∴ q

or

p v q
⌐q
∴ p

**5) Constructive Dilemma (C.D.)**

(p => q) Λ (r => s)
p v r
∴q v s

**6) Destructive Dilemma (D.D.)**

(p => q ) Λ (r => s)
⌐ q v ⌐ s
∴⌐ p v ⌐ r

**7) Simplification (Simp.)**

p Λ q
∴ p /q

**8) Conjunction (Conj.)**

p
q
∴ p Λ q

**9) Addition (Add.)**

p
∴ p v q

First six rules are standard rules of traditional logic. Last three rules need a little clarification. Consider, for example, simplification. Since p Λ q is given to us, we accept that p is true, and q is true as well. So there is no harm in dropping any of them. The case of conjunction is slightly different. p is given to us, so we take it as true; q is given to us. So we take q also as true. Since both are taken as true we can conveniently conjoin them. The case of addition, again, is different. Suppose that we have only p in the premises. Since it is a premise, we take it as true. Suppose that we require q to be added to p. We do not know whether q is true or not. There is no harm in adding q to p because even if q is false p v q still remains true because p is true. After all, one true component can make disjunction true. But what is important is that conjunction does not mean addition. In logical language, addition means disjunction but not conjunction.

The rest of our job is very easy; just apply relevant rules for relevant pairs of lines. It needs only practice and detective's eyes to identify relevant lines and the rule applicable to those lines.

All arguments, which are required to be tested, are valid only because these are proofs only for validity but not for invalidity.

## 1.4 TESTING THE VALIDITY OF ARGUMENTS

Let us begin with the argument we have seen above:

| | | |
|---|---|---|
| 1) | A => B | |
| 2) | B => C | |
| 3) | C => D | |
| 4) | ⌐D | |
| 5) | A v E / ∴ E | |
| 6) | A => C | 1, 2, H.S. |
| 7) | A => D | 6, 3, H.S. |
| 8) | ⌐A | 7, 4, M.T. |
| 9) | E | 5, 8, D.S. |

The original argument's premises are symbolized up to the fifth row. After the slant line is written the conclusion, which is to be proved through the formal proof. The way we have found out the sub-conclusion six (A => C) is written in the justification on right hand side, i.e., by applying the hypothetical syllogism (see above the rules of inference and their abbreviations) to the first and second premises (1, 2), we got the sixth premise or sub-conclusion. The rest of the steps can be easily understood through the justification written on the right hand side. Thus by applying elementary valid argument forms, we could derive the conclusion of the original argument and so this is a valid argument; we have established its validity by constructing a formal proof. Let us see certain formal proofs already constructed:

2

| | | |
|---|---|---|
| 1 | (B v N) => ( K Λ L) | |
| 2 | ⌐K | |
| 3 | ⌐M / ∴ ⌐B Λ ⌐M | |
| 4 | ⌐K v ⌐L | 2, Add. |
| 5 | ⌐(B v N) | 1, 4, M.T. |
| 6 | ⌐B Λ⌐ N | 5, De.M. |
| 7 | ⌐B | 6, Simpl. |
| 8 | ⌐B Λ ⌐M | 7,3, Conj. |

3

| | | |
|---|---|---|
| 1 | (K => A) Λ (M => D) | |
| 2 | ⌐A | |
| 3 | ⌐D / ∴ ⌐K Λ ⌐M | |
| 4 | K=> A | 1, Simpl. |
| 5 | ⌐K | 4,2, M.T. |
| 6 | M => D | 1, Simpl. |
| 7 | ⌐M | 6,3, M.T. |
| 8 | ⌐K Λ ⌐M | 5,7, Conj. |

4

1 (M v N) => (P Λ Q)

5

1 (A Λ B) => (C v D)

2 N / ∴ P

3 M v N 2, Add.

4 P Λ Q 1, 3, M.P.

5 P 4, Simpl.

2 A

3 B / ∴ C v D

4 A Λ B 2, 3, Conj.

5 C v D 1, 4, M.P.

6

1 ( T => K) Λ (R => S)

2 S => D

3 D => T

4 R / ∴ T

5 R => S 1 Simp.

6 S 5,4, M. P.

7 D 2, 6, M. P.

8 T 3, 7, M.P.

7

1 (A v B) Λ(⌐D ΛE)

2 A v B =>K / ∴ K Λ(⌐D ΛE)

3 A v B) 1, Simp.

4 K 2, 3, M.P.

5 ⌐D ΛE 1, Simp.

6 K Λ( ⌐D ΛE) 4, 5, Conj.

8

1 (P =>Q) Λ(R=>S)

2 ⌐A=> ⌐Q

3 A => ⌐B

4 B / ∴ ⌐P v ⌐S

5 ⌐A 3,4, M.T.

6 ⌐ Q 2,5, M.P.

7 P=>Q 1, Simp.

8 ⌐P 7, 6, M.T.

9 ⌐P v ⌐S 8, Add.

9

1 Av(B ΛC)

2 A=>P

3 ⌐P / ∴ C

4 ⌐A 2,3, M.T.

5 B ΛC 1,4, D.S.

6 C 5, Simp.

10

1 A Λ (B v C)

11

1 ⌐B

2 A=>P
3 Q / ∴ PΛQ
4 A 1, Simp.
5 P 2,4, M.P.
6 P Λ Q 5,3, Conj.

2 ¬D
3 (A=>B) Λ(C=>D)
4 K / ∴ C (K Λ¬A)
5 A=>B 3, Simp.
6 ¬A 5, 1, M.T.
7 C=>D 3, Simp.
8 ¬C 7, 2, M.T.

12

1 (B≡K)=>(Z ΛD)
2 ¬(Z ΛD) / ∴ ¬(B≡K)
3 ¬ (B≡K) 1,2, M.T.

13

1 (K ΛT)=>(A v B)
2 (A v B)=>(P Λ¬L)
3 (P Λ¬L)=>D
4 ¬(D) / ∴ ¬(K ΛT)
5 (KΛT)=>(P Λ¬L) 1,2, H.S.
6 (K ΛT)=>D 5,3, H.S.
7 ¬(K ΛT) 6,4 M.T.

14

1 (K Λ A)=>(¬ B v C)
2 M=>(K Λ A)
3 M / ∴ ¬B v C
4 M=> (¬B v C) 2,1. H.S.
5 ¬B v C 4,3, M.P.

15

1 A=>D
2 B=>C
3 A v B / ∴ D v C
4 (A=>D)Λ(B=>C) 1,2,Conj.
5 D v C 4,3, C.D.

16

1 A=>D
2 B=>C
3 ¬D v ¬C/ ∴ ¬A v ¬B
4 (A=>D)Λ(B=>C) 1,2,Conj.
5 ¬Av ¬B 4,3, D.D.

17

1 (A=>G)=>(K v ¬D)
2 ¬(K v ¬ D) / ∴ ¬(A=>G)
3 ¬(A=>G) 1,2, M.T.

18

1 JV(K Λ L)
2 J=>D
3 ¬D/ ∴ K Λ L
4 ¬J 2,3, M.T.
5 (K ΛL) 1,4 D.S.

19

1 D v (A=>B)
2 (A=>B)=>(C v K)
3 ¬(C v K) / ∴ D
4 ¬(A=>B) 2,3, M.T.
5 D 1,4, D.S.

20

1 A Λ(B=>C)
2 B / ∴ C
3 B=>C 1, Simp.
4 C 3,2, M.P.

21

1 (A=>B) Λ(C=>D)
2 A/ ∴ B v D
3 A v C 2, Add.
4 B v D 1,3, C.D.

22

1 A v (B ΛC)

23

1 A=>B

2 A=>D
3 ⌐D / ∴ B
4 ⌐A 2,3, M.T.
5 B ΛC 1,4 , D.S.
6 B 5, Simp.

2 B=>C
3 ⌐C / ∴ ⌐A
4 ⌐B 2,3, M.T.
5 ∴ ⌐A 1,4, M.T.

24

1 (A v B)=> C
2 D=> ⌐ C
3 D / ∴ ⌐(A v B)
4 ⌐C 2,3, M.P.
5 ⌐(A v B) 1,4, M.T.

25

1 (A=>C) Λ(B=>D)
2 K=>A
3 K /∴C v D
4 A 2,3, M.P.
5 A v B 4, Add.
6 C v D 1,5, C.D.

26

1 A v (B=>C)
2 A=>D
3 ⌐D /∴B=>C
4 ⌐A 2,3, M.T.
5 B=>C 1,4, D.S.

27

1 (A=>B)=>(C=>D)
2 (E=>F)=>(A=>B)
3 ⌐(C=>D) /∴⌐ (E=>F)
4 (E=>F)=>(C=>D) 2,1, H.S.
5 ⌐(E=>F) 4,3, M.T.

28

1 (K≡L) => (A Λ B)
2 D => (K≡L)
3 D / ∴A
4 D => (A Λ B) 1,2,H.S.
5 A Λ B 4,3,M.P.
6 A 5, Simp.

29

1 A Λ B
2 (A v C)=>D/∴(A ΛD)
3 A 1, Simp.
4 A v C 3, Add.
5 D 2,4, M.P.
6 (AΛD) 3,5, Conj.

30

1 I=>J
2 J=>K
3 L=>M
4 I v L /∴ K v M
5 I=>K 1,2, H.S.
6 (I=>K)Λ(L=>M) 5,3,Conj.
7 K v M 6,4, C.D.

31

1 (E v F) Λ(G v H)
2 (E=>G) Λ(F=>H)
3 ⌐G /∴H
4 E v F 1, Simp.
5 G v H 1, Simp.
6 H 5,3, D.S.

32

1 N=>(O ΛN)
2 (OΛ N)=>P
3 ⌐P /∴⌐N
4 ⌐ (OΛ N) 2, 3, M.T.

33

1 (W=>Y)=>Z
2 W=>Y /∴(Z v X)
3 Z 1,2,M.P.
4 Z v X 3, Add.

| | | |
|---|---|---|
| 5 | ⌐N | 1,4, M.T. |

34

| | | |
|---|---|---|
| 1 | (A v B)=>C | |
| 2 | (C v B)=>(A=> D) | |
| 3 | A ΛD /∴ (D v F) | |
| 4 | A | 3, Simp. |
| 5 | (A v B) | 4, Add. |
| 6 | C | 1,5, M.P. |
| 7 | (C v B) | 6, Add. |
| 8 | A=>D | 2,7, M.P. |
| 9 | D | 8,4, M.P. |
| 10 | (D v F) | 9, Add. |

35

| | | |
|---|---|---|
| 1 | F=> ⌐ G | |
| 2 | ⌐F=> (H=> G) | |
| 3 | (⌐I v ⌐H) => G | |
| 4 | ⌐I /∴H => G | |
| 5 | ⌐I v ⌐ H | 4, Add. |
| 6 | G | 3,5, M.P. |
| 7 | ⌐F | 1,6, M.T. |
| 8 | H=>G | 2,7, M.P. |

36

| | | |
|---|---|---|
| 1 | (L=>M)=>(N≡O) | |
| 2 | (P=> ⌐ Q)=>(M=> ⌐ Q) | |
| 3 | [(P=> ⌐ Q)v(R≡S)} Λ(N v O)]=>{(R≡S)=>(L =>M)} | |
| 4 | (P=> ⌐ Q)v(R≡S) | |
| 5 | N v O /∴ (M≡ ⌐ Q)v(N≡O) | |
| 6 | {(P=> ⌐ Q) v (R≡S)} Λ(N v O) | 4,5, Conj. |
| 7 | (R≡S) => (L=>M) | 3,6, M.P. |
| 8 | (R≡S) => (N≡O) | 7,1 H. S. |
| 9 | {(P=> ⌐ Q) => (M≡ ⌐ Q)} Λ{(R≡S) => (N≡O)} | 2, 8, Conj. |
| 10 | (M≡ ⌐ Q) v (N≡O) | 9,4, C.D. |

37

| | | |
|---|---|---|
| 1 | (F=>G) Λ(H=>I) | |
| 2 | J=>K | |
| 3 | (F v J) Λ(H v L) /∴G v K | |
| 4 | F=>G | 1, Simp. |
| 5 | (F=>G)Λ(J=>K) | 4,2,Conj. |
| 6 | F v J | 3, Simp. |
| 7 | G v K | 5,6, C.D. |

38

| | | |
|---|---|---|
| 1 | (⌐M Λ⌐N) => (O => N) | |
| 2 | N=>M | |
| 3 | ⌐M /∴⌐O | |
| 4 | ⌐N | 2,3, M.T. |
| 5 | (⌐M Λ⌐N) | 3,4, Conj. |
| 6 | O => N | 1,5, M.P. |
| 7 | ⌐O | 6,4, M.T. |

39

| | | |
|---|---|---|
| 1 | (K v L)=>(M v N) | |
| 2 | (M v N)=>(O ΛP) | |
| 3 | K /∴O | |
| 4 | K v L | 3, Add. |
| 5 | M v N | 1,4, M.P. |
| 6 | OΛP | 2,5, M.P. |
| 7 | O | 6, Simp. |

40

| | | |
|---|---|---|
| 1 | W => (W ΛX) | |
| 2 | (W ΛX) => (W ΛY) | |
| 3 | (W ΛY) =>Z /∴W=>Z | |
| 4 | W=> (W ΛY) | 1,2 H.S. |
| 5 | W=> Z | 4,3, H.S. |

41

| | | |
|---|---|---|
| 1 | A=>B | |
| 2 | C=>D | |

42

| | | |
|---|---|---|
| 1 | (E v F)=>(G ΛH) | |
| 2 | (G v H)=>I | |

| | | |
|---|---|---|
| 3 | A v C /∴B v D | |
| 4 | (A=>B) Λ (C=>D) | 1,2, Conj. |
| 5 | B v D | 4,3, C.D. |

| | | |
|---|---|---|
| 3 | E /∴I | |
| 4 | E v F | 3, Add. |
| 5 | G Λ H | 1,4, M.P. |
| 6 | G | 5, Simp. |
| 7 | G v H | 6, Add. |
| 8 | I | 2,7, M.P. |

43

| | | |
|---|---|---|
| 1 | J=>K | |
| 2 | K v L | |
| 3 | (L Λ⌐J)=>(M Λ⌐J) | |
| 4 | ⌐K /∴M | |
| 5 | L | 2,4, D.S. |
| 6 | -J | 1,4, M.T. |
| 7 | L Λ⌐J | 5,6, Conj. |
| 8 | M Λ⌐J | 3,7, M.P. |
| 9 | M | 8, Simp. |

44

| | | |
|---|---|---|
| 1 | (D v E) => (F ΛG) | |
| 2 | D /∴F | |
| 3 | D v E | 2, Add. |
| 4 | F Λ G | 1,3, M.P. |
| 5 | F | 4, Simp. |

45

| | | |
|---|---|---|
| 1 | Q=>R | |
| 2 | R=>S | |
| 3 | ⌐S /∴⌐Q Λ⌐R | |
| 4 | ⌐R | 2,3, M.T. |
| 5 | ⌐Q | 1,4, M.T. |
| 6 | ⌐Q Λ⌐R | 5,4, Conj. |

46

| | | |
|---|---|---|
| 1 | (T=>U) Λ(V=>W) | |
| 2 | (U=>X) Λ(W=>Y) | |
| 3 | T /∴X v Y | |
| 4 | T=>U | 1, Simp. |
| 5 | U | 4,3, M.P. |
| 6 | U=>X | 2, Simp. |
| 7 | X | 6,5, M.P. |
| 8 | X v Y | 7, Add. |

47

| | | |
|---|---|---|
| 1 | T => U | |
| 2 | V v ⌐U | |
| 3 | (⌐V Λ⌐W) /∴⌐T | |
| 4 | ⌐V | 3, Simp. |
| 5 | ⌐U | 2,4, D.S. |
| 6 | ⌐T | 1,5, M.T. |

48

| | | |
|---|---|---|
| 1 | ⌐X =>Y | |
| 2 | Z =>X | |
| 3 | ⌐X /∴Y Λ⌐Z | |
| 4 | ⌐Z | 2,3, M.T. |
| 5 | Y | 1,3 , M.P. |
| 6 | Y Λ⌐Z | 5,4, Conj. |

49

| | | |
|---|---|---|
| 1 | (A v B) => ⌐ C | |
| 2 | C v D | |
| 3 | A /∴D | |
| 4 | A v B | 3, Add. |
| 5 | ⌐ C | 1,4, M.P. |

50

| | | |
|---|---|---|
| 1 | (H => I) Λ(J => K) | |
| 2 | K v H | |
| 3 | ⌐K /∴I | |
| 4 | H | 2,3, D.S. |
| 5 | H=>I | 1, Simp. |

| | | |
|---|---|---|
| 6 | D | 2,5, D.S. |

| | | |
|---|---|---|
| 6 | I | 5, 4, M.P. |

51

| | | |
|---|---|---|
| 1 | L v (M => N) | |
| 2 | ⌐L=> (N => O) | |
| 3 | ⌐L /∴M => O | |
| 4 | N => O | 2,3, M.P. |
| 5 | M=> N | 1,3, D.S. |
| 6 | M => O | 5,4, H.S. |

52

| | | |
|---|---|---|
| 1 | K=>L | |
| 2 | M=>N | |
| 3 | (O=>N) Λ(P=>L) | |
| 4 | (⌐N v ⌐L) Λ(⌐M v ⌐ O)/ ∴ (⌐O v ⌐P) Λ(⌐M v ⌐ K) | |
| 5 | (M => N) Λ (K=>L) | 2,1, Conj. |
| 6 | ⌐N v ⌐L | 4, Simp. |
| 7 | ⌐M v ⌐ K | 5,6,D.D. |
| 8 | ⌐ O v ⌐P | 3,6, D.D. |
| 9 | (⌐O v ⌐ P) Λ (⌐M v ⌐ K) | 8,7.Conj. |

53

| | | |
|---|---|---|
| 1 | (G=> H) => ( I≡J) | |
| 2 | K v⌐ (L=> M) | |
| 3 | (G=> H) v ⌐K | |
| 4 | N=> (L=> M) | |
| 5 | ⌐ (I≡J) / ∴⌐N | |
| 6 | ⌐ (G => H) | 1,5, M.T. |
| 7 | ⌐K | 3,6, D.S. |
| 8 | ⌐(L=>M) | 2,7, D.S. |
| 9 | ⌐N | 4,8, M.T. |

54

| | | |
|---|---|---|
| 1 | (O => ⌐ P) Λ (⌐ Q => R) | |
| 2 | (S =>T) Λ (⌐U=> ⌐V) | |
| 3 | (⌐P => S) Λ (R => ⌐ U) | |
| 4 | (T v ⌐ V) => (W ΛX) | |
| 5 | O v ⌐ Q /∴ (W ΛX) | |
| 6 | ⌐P v R | 1,5, C.D. |
| 7 | S v ⌐ U | 3,6, C.D. |
| 8 | T v ⌐ V | 2,7, C.D. |
| 9 | W Λ X | 4,8, M.P. |

55

| | | |
|---|---|---|
| 1 | E => (F Λ ⌐ G) | |
| 2 | (F v G) => H | |
| 3 | E / ∴ H | |
| 4 | F Λ ⌐ G | 1,3, M.P. |
| 5 | F | 4, Simp. |
| 6 | F v G | 5, Add. |
| 7 | H | 2,6, M.P. |

## 1.5 TESTING OF THE VALIDITY OF ARGUMENTS (VERBAL)

For change, let us start with verbal form of argument and symbolize the statements and logical constants before proceeding to test the validity of the arguments. (Problems are worked out at the end.)

I ). If Rama joins, then the club's social prestige will rise; and if Krishna joins, then the club's financial position will be more secure. Either Rama or Krishna joins. If the club's social prestige rises, then Krishna will join; and if the club's financial position becomes more secure, then Govinda will join. Therefore either Krishna or Govinda will join.

II ). If Vishnu received the wire, then he took the plane; and if he took the plane, then he will not be late for the meeting. If the telegram was incorrectly addressed, then Vishnu will be late for the meeting. Either Vishu received the wire or the telegram was incorrectly addressed. Therefore either Vishnu took the plane or he will be late for the meeting.

III ). If Narayana buys the plot, then an office building will be constructed; whereas if Madhava buys the plot, then it quickly will be sold again. If Keshava buys the plot, then a store will be constructed; and if the store is constructed, then Lakshmi will offer to lease it. Either Narayana or Keshava will buy the lot. Therefore either an office building or a store will be constructed.

IV ). If Jagannath goes to the meeting, then a complete report will be made; if Jagannath does not go to the meeting, then a special election will be required. If a complete report is made, then an investigation will be launched. If Jagannath going to the meeting implies that a complete report will be made, then if the making of a complete report implies that an investigation will be launched, then either Jagannath goes to the meeting and an investigation is launched or Jagannath does not go to the meeting and no investigation is launched. If Jagannath goes to the meeting and an investigation is launched, then some members will have to stand trial. But if Jagannath does not go to the meeting and no investigation is launched then the organization will disintegrate very rapidly. Therefore either some members will have to stand trial or the organization will disintegrate very rapidly.

Statements are symbolized in the following manner:

1)

| | | |
|---|---|---|
| 1 | Rama joins | =R |
| 2 | The club's social prestige will rise | = S |
| 3 | Krishna joins | = K |
| 4 | The club's financial position rises | = F |
| 5 | Govinda will join | = G |

Now the argument becomes:

| | | |
|---|---|---|
| 1 | (R=> S) Λ (K=> F) | |
| 2 | R v K | |
| 3 | (S=> K) Λ (F=> G)/∴ K v G | |
| 4 | S v F | 1, 2, C.D. |
| 5 | K v G | 3, 4, C.D. |

Answer to the first argument makes one point very clear. Verbal expression is naturally very long and tedious, whereas symbolic representation is short and clear.

Subsequent examples are symbolized in a similar fashion

2)

| | | |
|---|---|---|
| 1 | Vishnu received the wire | =V |
| 2 | He took the plane | =P |
| 3 | He will not be late of the meeting | = ⌐L |
| 4 | Telegram was incorrectly addressed | = ⌐T |
| 5 | Vishnu will be late for the meeting | = L |

Now the arguments becomes:

1 (V=> P) Λ(P => ⌐ L)

| | | |
|---|---|---|
| 2 | (¬T=> L) | |
| 3 | V v ¬T | /∴P v L |
| 4 | V=> P | 1, Simp. |
| 5 | (V=> P) Λ (¬T=> L) | 4, 2, Conj. |
| 6 | P v L | 5, 3, C.D. |

The students are advised to construct formal proof of validity for the remaining arguments.

## 1.6 LET US SUM UP

Modern Logic is an extension of traditional logic. However, there is qualitative difference in testing. Difference consists in accuracy and clarity of proof. Nine rules of inference include many rules from traditional logic like modus ponens. Rulr of inference is applied to the whole line. All nine rules are not required always. Only some rules are required. There is no rule, which says that one line must be considered only once.

## 1.7 KEY WORDS

**Modus Ponens (MP) and Modus Tollens (MT)**: Modus ponens is a valid, simple argument form sometimes referred to as affirming the antecedent or the law of detachment. It is closely related to another valid form of argument, *modus tollens* or "denying the consequent."

**Validity**: An argument is valid if and only if the truth of its premises entails the truth of its conclusion. It would be self-contradictory to affirm the premises and deny the conclusion. The corresponding conditional of a valid argument states that logical truth and the negation of its corresponding conditional is a contradiction. The conclusion is a logical consequence of its premises.

**Polysyllogism**: it is a series of syllogisms in which the conclusion of the preceding one becomes the premise of the following syllogism. Sorites is one type of polysyllogism in which all the conclusions except the last are suppressed.

**Check Your Progress**

**Note:** a) Use the space provided for your answer.
b) Check your answers with those provided at the end of the unit.

There are 54 arguments, which have been examined. Just as you symbolized verbal form of argument, you substitute statements for symbols. Use your own statements to construct arguments for as many arguments as you can.

## 1.8 FURTHER READINGS AND REFERENCES

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## 1.9 ANSWERS TO CHECK YOUR PROGRESS

Clue: Symbolize each component using the first letter of first noun (proper or common) or verb.

3) Narayana buys the plot – N
Office building…… - O
Madhava buys the plot - M
It will be …….sold - S
Keshava buys…….. - K
…..store will be constructed –C
Lakshmi will ……. - L
Let us symbolize components accordingly:

1 N => O
2 M => S
3 K => C
4 C=> L
5 N v K / ∴ O v C
6 (N=>O) Λ (K => C) 1, 3, Conj.
7 ∴ O V C 6, 5, C.D.

It is quite obvious that lines 2 and 4 are not required to test the validity though they are parts of argument.

4) Jagannath goes to the meeting----- J
complete report----------- C
special election----------------- S
investigation will be launched----- I
some members will have to stand a trial---- M
the organization will disintegrate-------- D
Let us symbolize the components accordingly:

1 J => C
2 ¬J => S
3 C => I

4 { ( J => C) Λ (C => I) } => (J Λ I ) v (⌐J Λ ⌐I )
5 (J Λ I) => M
6 (⌐J Λ ⌐I) => D /∴ M v D
7 (J => C) Λ (C => I) 1, 3, Conj.
8 (J Λ I ) v (⌐J Λ ⌐I ) 4, 7, M. P.
9 {(J Λ I) => M } Λ {(⌐J Λ ⌐I) => D} 5, 6, Conj
10 ∴ M v D 9, 8, C.D..

# UNIT 2 FORMAL PROOF OF VALIDITY: RULES OF REPLACEMENT

**Contents**



## 2.0 OBJECTIVES

The main objective of this unit is to expose the inadequacy of the rules of inference. While this is the primary objective, which is intended to be achieved, there is another objective. It is to demonstrate that logic is a growing science. If new techniques of testing arguments are invented, then logic stands on par with technological science where continuous inventions and improvements are the order of the day. This unit also serves to demonstrate a crucial factor that all arguments do not fall under one or two categories. Therefore the same set of rules cannot guarantee success.

By the time you go through this unit, you should be in a position to identify the type of rules that are required to test given argument. This sort of ability can be acquired only by practice. Therefore, the arguments are designed in such a way that you are required to employ both the rules of inference and replacement.

## 2.1 INTRODUCTION

Not all arguments can be tested only with the rules of inference, though as shown in the previous unit, highly complex and diverse arguments succumb to these rules. Just as modern logic tried to supplement traditional logic, within modern logic, the need was felt to supplement the rules of inference. Hence we have the rules of replacement. The structure of argument may be such that it may require only the rules of replacement or only the rules of inference as we found it out in the previous unit or both. We have ten such rules, which are called the rules of replacement. The difference between these two sets of rules is that the rules of inference are themselves inferences whereas rules of replacement are not. However, the rules of replacement are restricted to change or change in the form of statements. For example, A or B is changed to B or A, or AΛ

(B v C) is changed to (A Λ B) v (AΛC). Also, in the mode of application of rules there is a restriction. Any rule of inference should be applied to the whole line only, as mentioned in the previous unit, whereas any rule of replacement can be applied to any part of the line. Suppose, for example, that a line consists of the expression 'A => (B Λ D)'. The consequent part cannot be simplified. The reason is simple. Suppose that either B or D is false, when A is false. Implication is still true. But we do not know whether B is true or D is true. On the other hand, no such restriction applies to any one of the ten rules, which are called replacement rules. All rules of replacement are logically equivalent expressions (i.e., these biconditionals (exposed with the symbol '≡' / '<=>' will be tautologies (true in all substitution instances) and so their replacement would be free from mistakes).

## 2.2 THE RULES OF REPLACEMENT

| | | |
|---|---|---|
| 1 | De Morgan's Law (De M.) | $\neg (p \wedge q) \equiv (\neg p \vee \neg q)$<br>$\neg (p \vee q) \equiv (\neg p \wedge \neg q)$ |
| 2 | Commutation Law (Com.) | $p \vee q \equiv q \vee p$<br>$p \wedge q \equiv q \wedge p$ |
| 3 | Double Negation(D.N.) | $\neg (\neg p) \equiv p$ |
| 4 | Transposition (Trans.) | $(p \Rightarrow q) \equiv (\neg q \Rightarrow \neg p)$ |
| 5 | Material Implication (Impl.) | $(p \Rightarrow q) \equiv \neg p \vee q$ |
| 6 | Material Equivalence (Equiv.) | $(p \equiv q) \equiv \{(p \Rightarrow q) \wedge (q \Rightarrow p)\}$<br>$(p \equiv q) \equiv \{(p \wedge q) \vee (\neg p \wedge \neg q)\}$ |
| 7 | Exportation (Exp.) | $\{(p \wedge q) \Rightarrow r\} \equiv \{p \Rightarrow (q \Rightarrow r)\}$ |
| 8 | Tautology (Taut.) | $p \equiv p \vee p$<br>$p \equiv p \wedge p$ |
| 9 | Association (Ass.) | $\{p \vee (q \vee r)\} \equiv \{(p \vee q) \vee r\}$<br>$\{p \wedge (q \wedge r)\} \equiv \{p \wedge q) \wedge r\}$ |
| 10 | Distribution (Dist.) | $\{p \wedge (q \vee r)\} \equiv \{(p \wedge q) \vee (p \wedge r)\}$<br>$\{p \vee (q \wedge r)\} \equiv \{p \vee q) \wedge (p \vee r)\}$ |

Some of these rules are structurally similar to some forms of immediate inference. For example, commutation law is similar, structurally, to simple conversion. Double negation is obversion. Transposition is what is called contraposition of hypothetical proposition in traditional logic. Finally, de Morgan's law is contradiction applied to disjunctive and conjunctive propositions.

Now our task is well defined. We examine, initially, arguments which require only these rules.

### a. TESTING THE VALIDITY OF ARGUMENTS (THE RULES OF INFERENCE AND REPLACEMENT)

1 $p \wedge q$

∴ ¬ (¬q v ¬p)

Ans:

1 p Λ q /∴¬ (¬q v ¬p)
2 q Λ p 1, Com.
3 ∴ ¬ (¬q v ¬p) 2, (De.M.)

2 1 p => q / ∴ ¬q => ¬p
∴ ¬q => ¬p 1, Trans.

3 1 ¬ p => q / ∴¬q => p
2 ¬q => ¬ (¬p) 1, Trans.
3. ∴ ¬q => p 2, D.N.

Let us use symbols for propositions instead of proposition form.

4 1 {I => (J => K)} Λ (J => ¬I) /∴ {(I Λ J) => K} Λ (J => ¬ I)}
2 ∴ {(I Λ J) => K} Λ (J => ¬I) 1, Exp.

5 1 (R Λ S) => (¬R v ¬S) /∴ (¬R v ¬S) => (R Λ S) .
2 ∴ (¬R v ¬S) => (R Λ S) 1,De.M.

6 1 (T v ¬U) Λ {(W Λ ¬V) => ¬T}/∴(T v ¬U) Λ {(W=>(¬V=>¬T)}
2 ∴(T v ¬U) Λ {(W=>(¬V=>¬T)} 1, Exp.

7 1 (X v Y) Λ (¬X v Z)/∴ (X v Y Λ ¬X) v {(X v Y) Λ Z}
2 ∴ (X v Y Λ ¬X) v {(X v Y) Λ Z} 1, Dist.

8 1 Z => (A => B) /∴Z => ¬{ ¬ (A => B)}
2 ∴Z => ¬{ ¬ (A => B)} 1, D.N.

9 1 (¬F v G) Λ (F => G) /∴F => G.)
2 (F => G) Λ (F => G) 1 Impl.
3 ∴ F=> G 2 Taut.

## 2.4 THE RULES OF INFERENCE AND REPLACEMENT

Now we shall consider different types of arguments, which may involve both kinds of rules.

Although construction of formal proof is an interesting section in Symbolic Logic, certain tips as regards its procedure is in order. 1. Concentrate on the general form of the argument, and not to he confused by the complexity of the statements involved. See the following example:

(A v D) => [(C v D) => (C => D)]
⌐ [(C v D) => (C => D)]
∴⌐ (A v D)

Although it is a highly complex argument verbally and symbolically, closer observation will tell us that it is an instance of Modus Tollens. 2. Simplification will help us in dropping the statements; H.S. will drop the middle term and connect with a new consequent. M.P. liberates the consequent. 3. Distribution enables us to transform a conjunction into disjunction and vice versa. Double negation avoids the negation signs. 4. If conclusion to be followed is a disjunction, it can be derived in three ways, i.e., by applying C.D. or D.D.; deduce a statement and the apply Addition, and find out an implication, then turn to a Disjunction. 5. If the conclusion is a conditional statement, it can be found through H.S., or deduce a disjunction and then turn it by applying Material implication. Thought-out application of the rules and imagination is the best means of success in constructing formal proof.

| | | | | |
|---|---|---|---|---|
| 10 | 1 | (O => ⌐ P) Λ (P => Q) | | |
| | 2 | Q => O | | |
| | 3 | ⌐R => P | / ∴R | |
| | 4 | ⌐ Q v O | 2, | Impl. |
| | 5 | O v ⌐ Q | 4, | Com. |
| | 6 | (O => ⌐ P ) Λ (⌐ Q => ⌐ P) | 1, | Trans, |
| | 7 | ⌐P v ⌐P | 6, 5, | C.D. |
| | 8 | ⌐P | 7, | Taut. |
| | 9 | ⌐ ⌐ R | 3, 8, | M.T. |
| | 10 | ∴ R | 9, | D.N. |

| | | | | |
|---|---|---|---|---|
| 11. | 1 | X => (Y => Z) | | |
| | 2 | X => (A => B) | | |
| | 3 | X Λ (Y v A) | | |
| | 4 | ⌐Z | / ∴B | |
| | 5 | (X Λ Y) => Z | 1, | Exp. |
| | 6 | (X Λ A) => B | 2, | Exp. |
| | 7 | (X Λ Y) v (X Λ A) | 3, | Dist. |
| | 8 | {(X Λ Y) => Z} Λ {(X Λ A) => B} | 5,6, | Conj. |
| | 9 | Z v B | 8, 7, | C.D. |
| | 10 | ∴B | 9, 4, | D.S. |

| | | |
|---|---|---|
| 12. | 1 | C => (D => ⌐ C) |

| | | | |
|---|---|---|---|
| | 2 | C ≡ D | / ∴ ⌐C v ⌐D |
| | 3 | C => (⌐ ⌐C => ⌐D) | 1, Trans. |
| | 4 | C => (C => ⌐D) | 3, D.N. |
| | 5 | (C Λ C) => ⌐D | 4, Exp. |
| | 6 | C => ⌐D | 5, Taut. |
| | 7 | ∴ ⌐C v ⌐D | 6, Impl. |

| | | | |
|---|---|---|---|
| 13. | 1 | E Λ (F v G) | |
| | 2 | (E Λ G) => ⌐ (H v I) | |
| | 3 | ⌐ (⌐H v ⌐I) => ⌐ (E Λ F) | / ∴H ≡ I |
| | 4 | (E Λ G) => (⌐H Λ ⌐I) | 2, De.M. |
| | 5 | ⌐ (H Λ I) => ⌐ (E Λ F) | 3, De.M. |
| | 6 | (E Λ F) => (H Λ I) | 5, Trans. |
| | 7 | {(E Λ F) => (H Λ I)} Λ {(E Λ G) => (⌐H Λ ⌐I)} | 6,4,Conj. |
| | 8 | (E Λ F) v (E Λ G) | 1, Dist. |
| | 9 | (H Λ I) v (⌐H Λ ⌐I) | 7,8, C.D. |
| | 10 | ∴ H ≡ I | 9, Equiv. |

| | | | |
|---|---|---|---|
| 14. | 1 | J v (⌐K v J) | |
| | 2 | K v (⌐J v K) / ∴ J ≡ K | |
| | 3 | (⌐K v J) v J | 1, Com. |
| | 4 | ⌐k v (J v J) | 3, Ass. |
| | 5 | ⌐K v J | 4, Taut. |
| | 6 | K => J | 5, Impl. |
| | 7 | (⌐J v K) v K | 2, Com. |
| | 8 | ⌐J v (K v K) | 7, Ass. |
| | 9 | ⌐J v K | 8, Taut. |
| | 10 | J => K | 9, Impl. |
| | 11 | (J => K) Λ (K => J) | 10, 6, Conj. |
| | 12 | ∴ J ≡ K | 11, Equi. |

| | | | |
|---|---|---|---|
| 15. | 1 | (E Λ F) Λ G | |
| | 2 | (F ≡ G) => (H v I) / ∴ I v H | |
| | 3 | E Λ (F Λ G) | 1, Ass. |
| | 4 | (F Λ G) Λ E | 3, Com. |
| | 5 | (F Λ G) | 4, Simp. |
| | 6 | (F Λ G) v (⌐ F Λ ⌐ G) | 5, Add. |
| | 7 | F ≡ G | 6, Equiv. |
| | 8 | H v I | 2, 7, M.P. |
| | 9 | ∴I v H | 8, Com. |

| | | | |
|---|---|---|---|
| 16. | 1 | (M => N) Λ (⌐ O v P) | |

| | | | |
|---|---|---|---|
| | 2 | M v ⌐ O | / ∴ N v P |
| | 3 | ∴ N v P | 1, 2,C.D. |
| 17. | 1 | (L v M) v (N Λ O) | |
| | 2 | (⌐ L Λ O) Λ ⌐ (⌐ L Λ M) | / ∴ ⌐L Λ N |
| | 3 | ⌐ L Λ [O Λ ⌐ (⌐ L Λ M)] | 2, Ass. |
| | 4 | ⌐ L | 3, Simp. |
| | 5 | L v {(M v (N Λ O)} | 1, Ass. |
| | 6 | M v (N Λ O) | 5,4, D.S. |
| | 7 | ⌐ (⌐ L Λ M) | 2, Simpl. |
| | 8 | Lv ⌐ M | 7, De. M. |
| | 9 | ⌐ M | 8, 4, D.S. |
| | 10 | N Λ O | 6, 9, D.S. |
| | 11 | N | 10, Simpl. |
| | 12 | ∴ ⌐ L Λ N | 4,11, Conj. |
| 18. | 1 | E => (F => G) | /∴ F => (E => G) |
| | 2 | (E Λ F) => G | 1, Exp. |
| | 3 | (F Λ E) => G | 2, Com. |
| | 4 | ∴F => (E => G) | 3, Exp. |
| 19. | 1 | H => (I Λ J) | / ∴H=>I |
| | 2 | ⌐ H v (I Λ J) | 1, Impl. |
| | 3 | (⌐ H v I) Λ (⌐ H v J) | 2, Dist. |
| | 4 | ⌐ H v I | 3, Simp. |
| | 5 | ∴ H => I | 4, Impl. |
| 20. | 1 | N => O | / ∴ (N Λ P) => O |
| | 2 | ⌐ N v O | 1, Impl. |
| | 3 | ⌐ P v ⌐ N v O | 2, Add. |
| | 4 | ⌐ (P Λ N) v O | 3, De.M. |
| | 5 | (P Λ N) => O | 4, Impl. |
| | 6 | ∴ (N Λ P) => O | 5, Com. |
| 21. | 1 | (Q v R) => S | / ∴Q => S |
| | 2 | ⌐ (Q v R) v S | 1, Impl. |
| | 3 | (⌐ Q Λ ⌐R) v S | 2, De.M. |
| | 4 | (⌐Q v S) Λ (⌐ R v S) | 3, Dist. |
| | 5 | ⌐Q v S | 4, Simp. |
| | 6 | ∴Q => S | 5, Impl. |

| | | | | |
|---|---|---|---|---|
| 22. | 1 | T => ⌐ (U => V) | / ∴T => U | |
| | 2 | T => ⌐ {⌐ ( U Λ ⌐V)} | 1, | D.N. |
| | 3 | ⌐T v (U Λ ⌐V) | 2, | Impl. |
| | 4 | (⌐T v U) Λ (⌐T v ⌐V) | 3, | Dist. |
| | 5 | ⌐T v U | 4, | Simp. |
| | 6 | ∴T => U | 5, | Impl. |
| 23. | 1 | W => (X v ⌐Y) | / ∴W => (Y => X) | |
| | 2 | W => (⌐Y v X) | 1, | Com. |
| | 3 | ∴W => (Y => X) | 2, | Impl. |
| 24. | 1 | H => (I v J) | | |
| | 2 | ⌐I | / ∴H => J | |
| | 3 | ⌐H v (I v J) | 1, | Impl. |
| | 4 | ⌐H v (J v I) | 3, | Com. |
| | 5 | (⌐H v J) v I | 4, | Ass. |
| | 6 | ⌐H v J | 5, 2, | D.S. |
| | 7 | ∴H=> J | 6, | Impl. |
| 25. | 1 | (K v L) => ⌐ (M Λ N) | | |
| | 2 | (⌐M v ⌐N) => (O ≡ P) | | |
| | 3 | (O ≡ P) => (Q Λ R) / ∴ (L v K) => (R Λ Q) | | |
| | 4 | (L v K) => ⌐ (M Λ N) | 1, | Com. |
| | 5 | (L v K) => (⌐M v ⌐N) | 4, | De.M. |
| | 6 | L v K) => (O ≡ P) | 5, 2, | H.S. |
| | 7 | (L v K) => (Q Λ R) | 6, 3, | H.S. |
| | 8 | ∴ (L v K) => (R Λ Q) | 7, | Com. |
| 26. | 1 | (D Λ E) => F | | |
| | 2 | (D => F) => G | /∴E=>G | |
| | 3 | (E Λ D) => F | 1, | Com. |
| | 4 | E => (D => F) | 3, | Exp. |
| | 5 | ∴E => G | 4, 2, | H.S. |
| 27. | 1 | (H v I) => {J Λ (K Λ L)} | | |
| | 2 | I | / ∴J Λ K | |
| | 3 | I v H | 2, | Add. |
| | 4 | H v I | 3, | Com. |
| | 5 | J Λ (K Λ L) | 1, 4, | M.P. |
| | 6 | (J Λ K) Λ L | 5, | Ass. |

7 ∴J Λ K 6, Simp.

28\. 1 (M v N) => (O Λ P)

2 ⌐O / ∴ ⌐M
3 ⌐O v ⌐P 2, Add.
4 ⌐ (O Λ P) 3, De.M.
5 ⌐ (M v N) 1, 4, M.T.
6 ⌐M Λ ⌐N 5, De.M.

7 ∴⌐M 6, Simp.

29\. 1 T Λ (U v V)
2 T => {U => (W Λ X)}

3 (T Λ V) => ⌐ (W v X) /∴W ≡ X
4 (T Λ U) => (W Λ X) 2, Exp.
5 (T Λ V) => (⌐W Λ ⌐X) 3, De.M.
6 {(T Λ U) => (W Λ X)} Λ {(T ΛV) => (⌐W Λ ⌐X)} 4, 5, Conj.
7 (TΛU) v (TΛV) 1, Dist.
8 (W Λ X) v (⌐WΛ⌐X) 6,7, C.D.

9 ∴W ≡ X 8, Taut.

30\. 1 Y => Z
2 Z => {Y => (R Λ S)}

3 ⌐ (R Λ S) /∴ ⌐Y
4 Y => {Y => (R Λ S)} 1,2, H.S.
5 (Y Λ Y) => (R Λ S) 4, Exp.
6 Y => (R Λ S) 5, Taut.

7 ∴⌐ Y 6, 3, M.T.

31\. 1 A v B

2 C v D / ∴{(A v B) Λ C} v {( (A v B) Λ D )
3 (A v B) Λ (C v D) 1, 2,Conj.

4 ∴ {(A v B) Λ C} v {(A v B) Λ D} 3, Dist.

32\. 1 (I v ⌐ ⌐J) Λ K

2 {⌐ L => ⌐ (K Λ J)} Λ {K => ( I=> ⌐M)} /∴ ⌐ (M Λ ⌐ L)
3 {(K Λ J) => L} Λ {K => (I => ⌐ M)} 2, Trans.
4 {(K Λ J) => L} Λ {(K Λ I) => ⌐ M} 3, Exp.
5 (I v J) Λ K 1, D.N.
6 K Λ (I v J) 5, Com.
7 (K Λ I) v (K Λ J) 6, Dist.
8 (K Λ J) v ( K Λ I) 7, Com.

| | | | |
|---|---|---|---|
| 9 | L v ⌐M | 4, 8, | C.D. |
| 10 | ⌐ M v L | 9, | Com. |
| 11 | ∴ ⌐ (M Λ ⌐ L) | 10, | De. M. |

## 2.5 TEST OF ARGUMENTS IN VERBAL FORM

Let us start with verbal form of argument and symbolize the statement and logical constants before proceeding to test the validity of the arguments. (Problems are worked out at the end.)

1. Oxygen in the tube either combines with filament to form an oxide or else it vanishes completely. Oxygen in the tube could not have vanished completely. Therefore the oxygen in the tube combined with the filament to form an oxide.
2. If a political leader who sees her former opinions to be wrong does not alter her course, she is guilty of deceit; and if she does alter her course, she is open to a charge of inconsistency. She either alters her course or she does not. Therefore either she is guilty of deceit or else she is open to a charge of inconsistency.
3. It is not the case that she either forgot or wasn't able to finish. She did not forget. Therefore she was able to finish.
4. She can have many friends only if she respects them as individuals. If she respects them as individuals, then she cannot expect them all to behave alike. She does have many friends. Therefore she does not expect them all to behave alike.
5. If the victim had money in his pockets, then robbery was not the motive for the crime. But robbery or vengeance was the motive for the crime. The victim had money in his pockets. Therefore vengeance must have been the motive for the crime.
6. Napoleon is to be condemned if he usurped power that was not rightfully his own. Either Napoleon was a legitimate monarch or else he usurped power that was not rightfully his own. Napoleon was not a legitimate monarch. So Napoleon is to be condemned.
7. If we extend further credit on the Wilkins account, they will have a moral obligation to accept our bid on their next project. We can figure a more generous margin of profit in preparing our estimates if they have a moral obligation to accept our bid on their next project. Figuring a more generous margin of profit in preparing our estimates will cause our general financial condition to improve considerably. Hence a considerable improvement in our general financial condition will follow from our extension of further credit on the Wilkins account.
8. Had Roman citizenship guaranteed civil liberties, then Roman citizens would have enjoyed religious freedom. Had Roman citizens enjoyed religious freedom, there would have been no persecution of the early Christians. But the early Christians were persecuted. Hence Roman citizenship would not have guaranteed civil liberties.
9. Jalaja will come if she gets the message provided that she is still interested. Although she did not come she is still interested. Therefore she did not get the message.
10. If the teller or the cashier had pushed the alarm button, the vault would have locked automatically and the police would have arrived within three minutes. Had the police arrived within three minutes, the robber's car would have been overtaken. But the robber's car was not overtaken. Therefore the teller did not push alarm button.

11. If people are always guided by their sense of duty, they forget the enjoyment of many pleasures; and if they are always guided by their desire for pleasure, they must often neglect their duty. People are either always guided by their sense of duty or always guided by their desire for pleasure. If people are always guided by their sense of duty, they do not often neglect their duty; and if they are always guided by their desire for pleasure, they do not forget for enjoyment of many pleasures. Therefore people must forget the enjoyment of many pleasures if and only if they do not often neglect their duty.

12. Although world population is increasing agricultural production is declining and manufacturing output remains constant. If agricultural production declines and world population increases, then either new food sources will become available or else there will be a radical redistribution of food resources in the world unless human nutritional requirements diminish. No new food sources will become available, yet neither will family planning be encouraged nor will human nutritional requirements diminish. Therefore there will be a radical redistribution of food resources in the world.

**Answers:** The components are symbolized in this way:

1.
| | | |
|---|---|---|
| 1 | Combined with filament: | C |
| 2 | Else it vanished: | V |
| 3 | Could not have vanished: | ⌐V |

Statements / Arguments:

1 C v V

2 ⌐V / ∴C 1, D. S.

2.
| | | |
|---|---|---|
| 1 | A political leader…does not alter her course: | ⌐C |
| 2 | She is guilty of deceit: | D |
| 3 | She alters her course: | C |
| 4 | She is open to a charge of inconsistency | I |

Statements:

1 (⌐ C => D) Λ (C => I)

2 C v ⌐ C / ∴D v I

3 ⌐ C v C 2, Com.

4 ∴D v I 1,3, C.D.

3.
| | | |
|---|---|---|
| 1 | It is not the case that: | ⌐ |
| 2 | She either forgets: | F |
| 3 | She did not forget. | ⌐ F |
| 3 | Not able to furnish: | ⌐A |

Statements / Arguments

1 ⌐ (F v ⌐A)

2 ⌐ F /∴ A

3 ⌐ F Λ A 1,De.M.

4 ∴ A 3,Simpl.

4. She does not respect them as individuals: ⌐ R
She can have many friends: F
She cannot expect… ⌐ E

Statement / Argument

1 ⌐R => ⌐ F
2 R => ⌐ E
3 F / ∴ ⌐ E
4 R 1, 3, M.T.
5 ∴ ⌐ E 2, 4, M.P.

5. 1 The victim had money… M
2 Robbery was not the motive ⌐R
3 Robbery or Vengence…. R v V

Statements / Argument

1 M => ⌐R
2 R v V
3 M / ∴V
4 ⌐ R 1, 3, M.P.
∴V 2, 4, D.S.

6. 1 He usurped power that was not…. U
2 Napoleon is to be condemned C
3 Napoleon was a legitimate… L

Statements / Argument

1 U => C
2 L v U
3 ⌐ L / ∴C
4 U 2,3, D.S.
∴ C 1, 4, M.P.

7. 1 We extend further credit… C
2 They will have a moral obligation…. M
3 We can figure…. F
4 Considerable improvement… I

Statement / Argument

1 C => M
2 M => F
3 F => I / ∴C => I
4 C => F 1,2, H.S.
∴C => I 4,3, H.S.

**Check Your Progress**

**Note:** a) Use the space provided for your answer.
b) Check your answers with those provided at the end of the unit.

Out of several problems, we have worked out seven problems. The student is advised to solve the rest, which is a very good method of learning to test the arguments of complicated structure.

## 2.6 LET US SUM UP

Just as the laws of traditional logic are inadequate to test the validity, rules of inference also are inadequate. So the stock of rules is further augmented with the help of the rules of replacement. Rule of inference applies to the whole line. However, the rule of replacement may apply to the whole line or any part of the line. Various types of arguments can be tested with the help of these rules.

## 2.7 KEY WORDS

**Usurp:** Usurp is to seize power from another, usually by illegitimate means.

**Technology:** Technology is a broad concept that deals with human's usage and knowledge of tools and crafts, and how it affects human's ability to control and adapt to environment. Technology is a term with origins in the Greek "*technologia,*" "*techne*" ("craft") and "*logia*" ("saying"). However, a strict definition is elusive; "technology" can refer to material objects of use to humanity, such as machines, hardware or utensils, but can also encompass broader themes, including systems, methods of organization, and techniques.

## 2.8 FURTHER READINGS AND REFERENCES


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## 2.9 ANSWERS TO CHECK YOUR PROGRESS

It may be noted that every symbol is the first letter first or second term in the respective component.

8 1) R => F
2) F => ⌐ C
3) C / ∴ ⌐ R
4) ⌐ F 2, 3, M.T.
5) ∴⌐ R 1, 4, M.T.

9 This particular argument is in need of some restructuring for the sake of convenience without changing meaning. It is done in the following manner.

If Jalaja will come and she is interested then she would have got the message. She did not come and she is interested. Therefore she did not get the message.
Now it is easy to symbolize.

1) (J Λ I) => S
2) ⌐ S Λ I / ∴ ⌐ J
3) J => (I =>S) 1,Exp.
4) J => (⌐ I v S) 3,Impl.
5) J => ⌐ (I Λ ⌐ S) 4,De.M.
6) I Λ ⌐S 2,Com.
7) ∴⌐J 5,6,M.T.

10

1) (T v C) => (V Λ P )
2) P => R
3) ⌐ R / ∴ ⌐ T
4) ⌐ P 2, 3, M.T.
5 ⌐ P v ⌐V 4, Add.
6 ⌐V v ⌐ P 5,Com.
7 ⌐ (T v C ) 1,6. M.T
8 ⌐ T Λ ⌐ C 7, De.M.
9 ∴ ⌐T 8, Simp.

11

1 (D => F) Λ (P => N)

2 D v P)

3 ( D => ⌐ N) Λ ( P => ⌐ F) / ∴ (F => ⌐ N) Λ (⌐ N => F)

4 F v N 1, 2, C.D.

5 ⌐ N v ⌐ F 3, 2, C. D.

6 ⌐ F => N 4, Impl.

7 N => ⌐ F 5,Impl.

8 ( ⌐ F => N) Λ ( N => ⌐ F ) 6, 7, Conj.

9 (⌐ N => F ) Λ ( F => ⌐N) 8, Trans.

10 ∴ ( F => ⌐ N ) Λ (⌐ N => F) 9, Com.

12

This argument also stands in need of restructuring of some sentences. It runs as follows. World population is increasing and agricultural production is declining and manufacturing output remains constant. When symbolized it becomes
W and A and M

Next the phrase 'unless human nutritional requirements diminish' becomes human nutritional requirements do not diminish'. And then the statement 'neither will family planning be encouraged nor will human nutritional requirements diminish' means the same as world population is increasing and human nutritional requirements do not diminish. The next stage is now obvious. The whole argument can be symbolized.

1 W Λ (A Λ M)
2 (A Λ W) => (N v R) Λ H
3 ⌐ N
4 W
5 ⌐ H / ∴R
6 (W Λ A) Λ M 1, Ass.
7 W Λ A 6, Simp.
8 (A Λ W) 7, Com.
9 (N v R) Λ H 2, 8, M.P.
10 N v R 9, Simp.
11 ∴ R 10, 3, D.S.

Note that line 5 is redundant though it is the part of the argument. Therefore it can be ignored.

## UNIT 3 CONDITIONAL PROOF AND INDIRECT PROOF

**Contents**



## 3.0 OBJECTIVE

In this unit I propose to introduce a new list of techniques of testing the validity of arguments. There are as many kinds of techniques as there are arguments. The main purpose of this unit is to make you understand that there is not a single technique which helps you to solve all kinds of problems.

It is not sufficient if you know the art of testing validity only. Therefore one of the aims is to introduce you to the art of testing invalidity also. To have a satisfactory knowledge of good argument you should also know what makes an argument bad. Therefore this unit introduces you to this aspect of the study of logic.

## 3.1 INTRODUCTION

The method of Conditional Proof (C.P.) is different in kind from the rules of inference or replacements. There are a certain types of arguments, which cannot be tested with any of the rules discussed in the previous chapters without further support.The rules discussed earlier are restricted only to those arguments, which have unconditional conclusions. So an argument, which has conditional conclusion, falls out of their purview. The most familiar example for conditional proposition is implicative proposition. Since implicative propositions have equivalent disjunctive and negation forms, they are also to be regarded as conditional propositions. Again, C.P is not a system of proof, which does away with the nineteen rules. Only, the number increases to twenty. Among them one rule is compulsorily used to test the validity when the conclusion is conditional. This rule is characteristic of C.P in the sense that nowhere else it is used. Hence this rule can be designated as the rule of C.P.

## 3.2 CONDITIONAL PROOF

Any deductive argument, whether valid or invalid, can be expressed in the form of a conditional proposition. What is more important to know is that the original argument is valid only when the corresponding conditional statement fulfills a condition known as 'tautology'. Otherwise the argument is invalid. Consider this example:

1). All A are B

All B are C / ∴ All A are C

Its corresponding conditional form is as follows:

"If all A are B and all A are C, then all A are C". (1).

Let the first premise be symbolized as $P_1$ and second as $P_2$. Conclusion is symbolized as C. Now (1) becomes:

$(P_1 \wedge P_2) \Rightarrow C$ (2)

(2) is said to be tautologous because its corresponding proposition form is tautologous. A proposition form is said to be tautologous when it has only true substitution. No matter how many substitutions we make for proposition form, all of them must be true. In other words, if there are 'n' number of instances in which substitution is made to the proposition form, then in all these 'n' instances the proposition form must be true. There are two conditions to be satisfied if C. P. should show that the argument is valid.

1). Conclusion must be a conditional proposition.

2). It should be possible to deduce a conditional proposition from a conjunction of premises by a sequence of elementary valid arguments which satisfy the relevant rules of inference. That is, all premises in C.P. should be supported by these rules. The additional premise, which is a characteristic mark of C.P., is always the antecedent of the conclusion and the construction of proof always begins with antecedent of the conclusion as the premise. This premise itself is called C.P. An example of argument, which requires C.P., is given below.

(3) P => (A=> B)

When P stands for the conjunction of premises, one of the rules of replacement, i.e., exportation rule permits us to rewrite (3) as:

(4) (PΛ A) => B

It is obvious that the conclusion of (4) is the consequent of the conclusion of (3). Since we start with an assumed premise, the proof is known as C.P. Here is the difference. All other premises are taken as true. The assumption should not really matter. Even if the assumed premise is false, it is possible to deduce valid conclusion. If B can be validly drawn from P and A then not only (A) is valid its corresponding original argument (3) also must be valid because (3) and (4) are logically equivalent argument of this form.

1. 1. (A v B)=> (CΛ D)

2). (D v E) => F / ∴ A=>F

We should start from assuming A.

3). A /∴ F C. P.

In C. P. always the first line must have this structure. Slash against line 3 in, ∴ and (C.P) indicate that the method of conditional proof is being used.

4). A v B 3, Add.
5). C Λ D 1, 4, M.P.
6). D 5, Simp.
7). D v E 6, Add.
8). ∴ F 2, 7, M.P.

If there is only one condition in the conclusion, then C.P is used once. If there are two conditions in the conclusion, then C.P. is used twice. In such cases the procedure to be followed is as follows.

2. 1). A=> (B=> C)
2). B=> (C=> D) /∴ A => (B =>D)
3). A /∴ B => D (C.P.)
4). B /∴ D (C.P.)
5). B => C 1, 4, M.P.
6). C 5, 4, M.P.
7). C => D 2, 4, M.P.
8). ∴D 7, 6, M.P.

## 3.3 INDIRECT PROOF (I.P.)

This method is also known as *reductio ad absurdum*, a method very common in the construction of proof of geometrical theorems. This method is characterized by a special feature. In order to prove a certain statement, its contradiction is assumed to be true from which the conclusion, which contradicts our assumption, is logically deduced. If A contradicts ⌐ B, then either A must be false or ⌐ B must be false. A cannot be false because it is logically deduced from what is purported to be true. Therefore ⌐B must be false, which means that B must be true. This is how a theorem in geometry or an argument in logic is, sometimes, proved.

This method has a distinct advantage. Sometimes the length of proof is too long. In logic it is important that we use the least number of steps. Second requirement is clarity. Combination of these two is what is most desired. In such circumstances, this method is most useful. The use of this method consists in beginning with the contradiction of what is to be proved. A point to be noted here is that, the contradiction of what has to be proved is marked by writing I.P. on the right hand side just adjacent to the assumption. In C.P. also we begin with assumption. The difference is that in the latter what is assumed is a part of the argument whereas in the case of former it is not. Consider this argument.

3.

1. A => (BΛC)
2. (B v D) => E
3. D v A /∴ E
4. ⌐ E I.P.
5. ⌐B Λ ⌐ D 2, 4, M.T.
6. ⌐D 5, Simp.

| | | | |
|---|---|---|---|
| 7. | A | 3, 6, | D.S. |
| 8. | BΛC | 1, 7, | M.P. |
| 9. | B | 8, | Simp. |
| 10. | B v D | 9, | Add. |
| 11. | E | 2, 10, | M.P. |
| 12. | E Λ ⌐ E | 11, 4, | Conj. |

Tenth Step can also be written and consequent step in this manner

| | | | |
|---|---|---|---|
| 13. | ⌐ B | 5, | Simp. |
| 14. | B Λ ⌐ B | 9, 13, | Conj. |

Whether we get E Λ ⌐ E or B Λ ⌐ B, the result remains the same. In both the cases there are steps in the argument whose conjunction leads to contradiction. Wherever there is contradiction, one conjunct must be false so that the other one has to be true.

## 3.4 THE STRENGHTHENED RULE OF CONDITIONAL PROOF

In Conditional Proof method, the conclusion depends upon the antecedent of the conclusion. There is another method, which is called the strengthened rule of conditional proof. In this method, the construction of proof does not necessarily assume the antecedent of the conclusion. The structure of this method needs some elaboration. An assumption is made initially. There is no need to know the truth-status of the assumption because an assumption may be false, but the conclusion can still be true. Further, the assumption can be any component of any premise or conclusion. This method is called the strengthened rule because we enjoy more freedom in making assumption or assumptions, which means that plurality of assumptions is allowed. It strengthens our repertoire of testing equipments. In this sense, this method is called the strengthened rule of C.P. Another feature of this method is the limit of assumption. The last step is always outside the limits of assumption. If there are two or more than two assumptions in an argument, then there will be a distinct last step with respect to each assumption. This last step can be regarded as the conclusion relative to that particular assumption. It shows that the last step is deduced with the help of assumption in conjunction with the previous steps in such a way that the rules of inference permit such conjunction. Before the conclusion is reached the function of assumption also ceases. Then it will have no role to play. Then, automatically, the assumption is said to have been discharged. When the strengthened rule of C. P. is used adjacent to the line of assumption, the word assumption is not mentioned unlike in the case of C.P. here the head of the bent arrow points to 'assumption'. In case of the strengthened rule of C.P., the conclusion is always a conditional statement which consists of statements from the sequence itself.

Thus the range of the application of condition is defined. In order to easily identify the range of its application, a slightly different method is used. An arrow is used to indicate what is assumed and with the help of the same arrow its range also is defined. The application of C.P. in restricted to the space covered by the arrows. All steps, which are outside this arrow, are also independent of the condition. While the head of the arrow marks the assumption, its terminus separates the lines, which depend upon the condition from the line, which does not depend on the condition. Since the conclusion does not depend upon its own antecedent, it has to depend upon the first premise only. In this sense, it is a strengthened condition. In this case there is no reason to mention C.P. because the arrow helps us to identify the assumption. Consider this example:

| | | | |
|---|---|---|---|
| 1. | (A v B) =>{(C v D) => E} /∴ A=> [(C Λ D) => E] | | |
| 2. | A | | |
| 3. | A v B | 2, | Add. |
| 4. | (C v D) => E | 1, 3, | M.P. |
| 5. | (CΛD) | | |
| 6. | C | 5, | Simp. |
| 7. | C v D | 6, | Add. |
| 8. | E | 4, 7, | M.P. |
| 9. | (C v D) => E | 5, 8, | C.P. |
| 10. | A => [CΛD) => E] | 2, 9, | C.P. |

Rules mentioned on the RHS make it clear that all lines from 3 to 9 depend on A either directly or through bent arrows. In lines 9 and 10 implication makes them C.P.

One advantage of C.P. in its strengthened form is that it has an extended application. It can be used in all those cases where conclusions are conditional, but do not appear to be so.

## 3.5 PROVING INVALIDITY

Unlike validity, invalidity is not governed by any rules. Of course, it is more than obvious that errors do not have any rules, which govern. On the other hand, only violation of a rule or rules makes arguments invalid. Hence the method of proving invalidity is different. The principle of inference dictates that a true premise and a false conclusion together result in invalidity. Therefore in order to determine invalidity we should assign truth-values in such a way that the premise or premises are true and the conclusion is false. If we succeed in doing so then the argument is invalid. This method is so simple that the test can be completed in one line as it happens in the case of truth-table. Let us consider some examples.

1. E=> (F v G)
2. G => (H Λ I)
3. ⌐ H /∴E => I

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 1 |
| E | F | G | H | I | ⌐H |

While following this method '0' should be assigned to the conclusion making the premises true. If this combination cannot be achieved, then the argument is valid, i.e., even after making the conclusion 0 if the premises cannot take the value 1, then the argument is valid. The components of conclusion and premises should be paired properly to carry out the test.

1. J => (K => L)
2. K=> (⌐ L=> M)
3. (L v M) => N /∴ J => N

| | | | | | |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 |

J K L ¬L M N

Here the conclusion is ‘0’ whereas the combination of premises is 1. Hence the argument is invalid.

1.

## 3.6 EXERCISES

I Here some arguments are given which are tested using the method of C. P.

1

1. P Λ (Q => R) / ∴ (P => Q) => (P => R)
2. P => Q / ∴ P => R C.P.
3. P / ∴ R C.P.
4. (P=> Q) => R 1, Exp.
5. ∴ R 4, 2, M.P.

2

1. P => (Q => R) / ∴ Q => (P => R)
2. Q / ∴ P => R C.P.
3. P / ∴ R C.P.
4. Q => R 1, 3, M.P.
5. ∴ R 4, 2, M.P.

’

3 1 A=> B / ≵ (B => C) => (A => C)
2 B => C /≵ A => C C.P.
3 A /≵ C C.P.
4 B 1, 3, M.P.
!!!6!≵ C 2, 4, M.P.

5

1. (A => B) Λ (A => C) /≵ A => (B v C)
2. A /≵ B v C C.P.
3. A=> B 1, Simp.
4. B 3, 2, M.P.
5. A=> C 1, Simp.

| | | | |
|---|---|---|---|
| 6. | C | 5, 2, | M.P. |
| 7. | ≹ B v C | 4, | Add. |

6

| | | | | |
|---|---|---|---|---|
| 1. | (A=> B) Λ (A=> C) | /A=> (BΛC) | | |
| 2. | A | /≹ B Λ C | | C.P. |
| 3. | A => B | | 1, | Simp. |
| 4. | B | | 3, 2, | M.P. |
| 5. | A => C | | 1, | Simp. |
| 6. | C | | 5, 2, | M.P. |
| 7. | ≹ B Λ C | | 4, 6, | Conj. |

7

| | | | | |
|---|---|---|---|---|
| 1. | (A => B) | /≹ (AΛC) => (BΛC) | | |
| 2. | A Λ C | /≹ B Λ C | | C.P. |
| 3. | A | | 2, | Simp. |
| 4. | B | | 1, 3, | M.P. |
| 5. | C | | 2, | Simp. |
| 6. | ≹ B Λ C | | 4, 5, | Conj. |

8

| | | | | |
|---|---|---|---|---|
| 1. | B => C | /≹ (A v B) => (C v A) | | |
| 2. | A v B | /≹ C v A | | C.P. |
| 3. | ⌐ A=> B | | 2, | Impl. |
| 4. | ⌐ A=> C | | 3, 1, | H.S. |
| 5. | A v C | | 4, | Impl. |
| 6. | ≹ C v A | | 5, | Com. |

9

| | | | | |
|---|---|---|---|---|
| 1. | (A v B) => C | /≹ [(C v D => E] => (A => E) | | |
| 2. | (C v D) => E | /≹ A => E | | C.P. |
| 3. | A | /≹ E | | C.P. |
| 4. | A v B | | 3, | Add. |
| 5. | C | | 1, 4, | M.P. |
| 6. | C v D | | 5, | Add. |
| 7. | ≹ E | | 2, 6, M.P. | |

II Here are some arguments, which can be proved using indirect method.

1.

| | | | |
|---|---|---|---|
| 1. | Av (B ΛC) | | |
| 2. | A => C /∴ C | | |
| 3. | ¬ C | | I.P. |
| 4. | ¬A | 2, 3, | M.T. |
| 5. | B Λ C | 1, 4, | D.S. |
| 6. | C | 5, | Simp. |
| 7. | C Λ ¬ C | 6, 3, | Conj. |

Seventh step involves contradiction; therefore ¬C is false which means that C is true.

2.

| | | | |
|---|---|---|---|
| 1. | (D v E) => (F => G) | | |
| 2. | (¬ G v H) => (DΛF) /∴ G | | |
| 3. | ¬ G | | I.P. |
| 4. | ¬ G v H | 3, | Add. |
| 5. | D Λ F | 2, 4, | M.P. |
| 6. | D | 5, | Simp. |
| 7. | D v E | 6, | Add. |
| 8. | F=> G | 1, 7, | M.P. |
| 9. | ¬ F | 8, 3, | M.T. |
| 10. | F | 5, | Simp. |
| 11. | F Λ ¬ F | 10, 9, | Conj. |

Eleventh step is contradiction. Therefore ¬G is false; which means that G is true.

3.

| | | | |
|---|---|---|---|
| 1. | (H => I)Λ(J => K) | | |
| 2. | (I v K) => L | | |
| 3. | ¬ L /∴ ¬ (H v J) | | |
| 4. | H v J | | I.P. |
| 5. | I v K | 1, 4, | C.D. |
| 6. | L | 2, 5, | M.P. |
| 7. | L Λ ¬ L | 6, 3, | Conj. |

7th step involves contradiction. Therefore H v J is false; which means that ¬ (H v J) is true.

4.

| | | | |
|---|---|---|---|
| 1. | (M v N) => (OΛP) | | |
| 2. | (O v Q) => (¬ R Λ S) | | |
| 3. | (R v T) => (M Λ N) /∴ ¬ R | | |
| 4. | R | | I.P. |
| 5. | R v T | 4, | Add. |
| 6. | M Λ N | 3, 5, | M.P. |
| 7. | O Λ P | 1, 6, | M.P. |
| 8. | O | 7, | Simp. |

| | | |
|---|---|---|
| 9. O v Q | 8, | Add. |
| 10. (⌐ R Λ S) | 2, 9, | M.P. |
| 11. ⌐ R | 10, | Simp. |
| 12. R Λ ⌐ R | 4, 11, | Conj. |

Twelfth step involves contradiction. Therefore R is false which means that ⌐R is true.

5.

| | | |
|---|---|---|
| 1. (V => ⌐ W) Λ (X =>Y) | | |
| 2. (⌐W => Z) Λ (Y => ⌐ A) | | |
| 3. (Z=> ⌐ B) Λ (⌐ A=> C) | | |
| 4. V Λ X / ≵ ⌐ B Λ C | | |
| 5. ⌐ (⌐ B Λ C) | | I.P. |
| 6. B v ⌐ C | 5, | De.M. |
| 7. ⌐ Z v A | 3, 6, | D.D. |
| 8. W v ⌐ Y | 2, 7, | D.D. |
| 9. ⌐V v ⌐X | 1, 8, | D.D. |
| 10. (V Λ X) Λ (⌐ V v ⌐ X) | 4, 9, | Conj. |

10th Step involves contradiction. Therefore ⌐ (⌐BΛC) is false, which mean that ⌐BΛC is true. We can also prove these arguments using formal proof of validity. Consider 3rd argument.

6.

| | | |
|---|---|---|
| 1. (H => I) Λ (J => K) | | |
| 2. (I v K) => L | | |
| 3. ⌐L / ≵ !⌐ (H Λ J) | | |
| 4. ⌐I Λ ⌐ K | 2, 3, | M.T. |
| 5. ⌐ I | 4, | Simp. |
| 6. ⌐ I v ⌐ K | 5, | Add. |
| 7. ⌐ H v ⌐ J | 1, 7, | D.D. |
| 8. ≵ ⌐ (H Λ J) | 8, | De.M. |

When the 3rd argument was solved using IP method, it involved 7 steps, whereas formal proof required 8 steps. Therefore the former is shorter and preferable.

Now consider the fifth agreement.

7

| | | |
|---|---|---|
| 1. (V => ⌐ W) Λ (X =>Y) | | |
| 2. (⌐W => Z) Λ (Y => ⌐A) | | |
| 3. (Z => ⌐ B) Λ (⌐A => C) | | |
| 4. V Λ X /≵ ⌐ B Λ C | | |
| 5. V=> ⌐ W | 1, | Simp. |
| 6. V | 4, | Simp. |
| 7. ⌐ W | 5, 6, | M.P. |
| 8. X => Y | 1, | Simp. |

| | |
|---|---|
| 9. X | 4, Simp. |
| 10. Y | 8, 9, M.P. |
| 11. ⌐ W => Z | 2, Simp. |
| 12. Z | 11, 7, M.P. |
| 13. Y => ⌐ A | 2, Simp. |
| 14. ⌐ A | 13, 10, M.P. |
| 15. Z => ⌐ B | 3, Simp. |
| 16. ⌐ B | 15, 12, M.P. |
| 17. ⌐A => C | 3, Simp. |
| 18. C | 17, 14, M.P. |
| 19. ∴ ⌐ B Λ C | 16, 18, Conj. |

When the 5th argument was solved using I.P. method, it involved 10 steps; whereas formal proof required 19 steps. Therefore the former is shorter and preferable.

III Using the method of reductio ad absurdum (Indirect Proof) the following are proved to be tautologies.

1

| | |
|---|---|
| 1 (A => B) v (⌐ A => B) | |
| 2 ⌐ {(A => B) v (⌐ A => B)} | 1, I. P. |
| 3 ⌐ (A => B) ∧ ⌐ (⌐ A => B) | 2, De. M. |
| 4 ⌐ (A => B) | 3, Sim. |
| 5 A ∧ ⌐ B | 4, De. M. |
| 6 ⌐ (⌐ A => B) | 2, Simp. |
| 7 ⌐ (A v B) | 6,Impl. |
| 8 A | 5, Simp. |
| 9 ⌐A ∧ ⌐ B | 7, De. M. |
| 10 ⌐A | 9, Simpl. |
| 11 A ∧ ⌐ A | 8, 10., Conj. |

Eleventh step involves contradiction which means that there is error in the second step, i.e., assumption. Therefore the given expression is a tautology.

2.

| | |
|---|---|
| 1 (A => B) v (B => A) | |
| 2 ⌐ {(A => B) v (B => A)} | 1, I. P. |
| 3 ⌐ (A => B) ∧ ⌐ (B => A) | 2, De. M. |
| 4 ⌐ (A => B) | 3, Simp. |
| 5 ⌐(⌐ A v B) | 4, Impl. |
| 6 ⌐ (B => A) | 3, Simp. |
| 7 ⌐(⌐ B v A) | 6, Impl. |
| 8 A ∧ ⌐ B | 5, De.M. |
| 9 A | 8, Simp. |
| 10 B ∧ ⌐ A | 7, De.M. |
| 11 ⌐ A | 10, Simp. |
| 12 A ∧ ⌐ A | 9,11, Cong. |

Explanation for this argument is the same as the one given to the previous one.

3

1 (A => B) v (B => C)
2 ⌐ {(A => B) v (B => C)} 1, I. P.
3 ⌐ (A => B) ∧ ⌐ (B => C) 2, De. M.
4 ⌐ (⌐A v B) ∧ ⌐ (⌐ B v C) 3, Impl.
5 (A ∧ ⌐ B) ∧ (B ∧ ⌐ C) 4, De.M.
6 A ∧ ⌐ B 5, Simp.
7 ⌐ B 6, Simp.
8 B ∧ ⌐ C 5, Simp.
9 B 8, Simp.
10 B ∧ ⌐ B 9,7, Conj.

Since nineth step involves contradiction, there is error in the second step. Therefore our assumption is wrong which means that the first step is a tautology.

4.

1 A v (A => B)
2 ⌐ {A v (A => B)} 1, I. P.
3 ⌐ A ∧ ⌐ (A => B) 2, De. M.
4 ⌐ A ∧ ⌐ (⌐ A v B) 3, Impl.
5 ⌐ A ∧ (A ∧ ⌐ B) 4, De. M.
6 (⌐ A ∧ A) ∧ ⌐ B 5, Ass.
7 ⌐ A ∧ A 6, Simp.

In this argument there is contradiction in the last step. Therefore the assumption is false. Therefore 1 is a tautology.

5.

1 P ≡ ⌐ ⌐ P
2 ⌐ (P ≡ ⌐ ⌐ P) 1, I. P.
3 ⌐ {(P => ⌐ ⌐ P) ∧ (⌐ ⌐ P => P)} 2, Equiv.
4 ⌐ {(P => P) ∧ (P => P) } 3,D.N.
5 ⌐ {(⌐ P v P) v (⌐ P v P)} 4, Impl.
6 (P ∧ ⌐ P) ∧ (P ∧ ⌐ P) 5, De.M.

In this argument there is contradiction in the last step. Therefore the assumption is false. Therefore 1 is a tautology.

6

1 ⌐ {(A => ⌐ A) ∧ (⌐ A => A)}
2 ⌐ [⌐ {(A => ⌐ A) ∧ (⌐ A => A)}] 1, I. P.
3 {(A => ⌐ A) ∧ (⌐ A => A)} 2, D. N.
4 (⌐ A v ⌐ A) ∧ (A v A) 3, Impl.
5 ⌐ A v ⌐ A ≡ ⌐ A By Taut.
6 A v A ≡ A By Taut.
7 A ∧ ⌐ A 6,5, Conj.

In this argument there is contradiction in the last step. Therefore the assumption is false. Therefore 1 is a tautology.

7. The next argument is very different.

1 ⌐ {(A => ⌐ A) v (⌐ A => A)}
2 (A => ⌐ A) v (⌐ A => A) 1, I. P.
3 (⌐ A v ⌐ A) v (A v A) 2, Impl.
4 ⌐ A v A By Taut.

.

It is important to note that the fourth step is not a contradiction. On the other hand, it itself is a tautology. It means that the line no. 1 is itself a contradiction.

### V. Truth-table technique and reductio ad absurdum method - a joint venture:

We can also prove the validity of an argument by integrating the method of reductio ad absurdum with the truth-table technique. We have to make certain assumptions before we use the combination of these two. These assumptions are as follows:

1. All premises are necessarily true. When the premises are truth-functionally compound, the truth-values of components should be such that the compound proposition is necessarily true.
2. The conclusion is necessarily taken to be false. When the conclusion is truth-functionally compound, the truth-values of components should be such that the conclusion is necessarily false.

While assigning the truth-values, in accordance with these assumptions, if we discover that any component takes the values 1 and 0 simultaneously, then it means that the path has led us to contradiction. Therefore the assumption that the argument is invalid is false. Hence it must be valid. What is important is that once a certain truth-value is assigned to a component, it becomes a permanent fixture of that component throughout the course of the argument. Let us consider this argument.

1

1. P1(A => B) => (C Λ⌐ D)
2. P2 (D => E) => F / ∴ ⌐ A => F
   Let us assume that (⌐ A => F) = 0
   (i.e., it is not the case that ⌐ A => F)
   This is possible only when ⌐ A=1 and F=0.
3. In P2 F= 0.
4. P2=1 iff (if and only if)
   (D => E) => F
   0 1 0
5. (D => E) = 0 iff (D => E)
   1 0 0
6. ⌐ D = 0 ∵ D = 1
7. (C Λ⌐ D) = 0 ∵ ⌐ D = 0; and if any one conjunct is false, then the whole conjunction is false.
8. When (CΛ⌐ D) = 0, which is the consequent, P1 can take the value 1 iff the antecedent (A => B) = 0 ∵ the consequent is false.

9. A = 0 ∵ ⌐ A = 1 (according to the law of contradiction, when A= 0, ⌐ A=1) (See2).
10. A => B necessarily takes the value 1 irrespective of the truth-value of B because A = 0 (See9).
11. 8 and 10 contradict.
12. (1), i.e., ⌐ (⌐ A => F) = 0 is false
13. ∴ ⌐ A => F

When P1, P2 and the conclusion are connected properly, it becomes a tautology. In order to get such an expression, implication should connect the conclusion to the premises which in turn are connected with conjunction.

Since the method of reductio ad absurdum demands that the conclusion must be assumed to be false when the given argument is valid, the truth-conditions of compound proposition must scrupulously be followed. Therefore if the conclusion is disjunctive, then both the components of the disjunction must be assigned 0-value. On the other hand, if the given conclusion is a conjunction, then it is sufficient if any one compound is assigned the 0-value. Thirdly, if the conclusion is the negation of conjunction, then the conjunction itself must be assigned the value-1, which means that both components of the conclusion must take the value-1.

Let us consider an argument in which conclusion is a conjunction.

2

P1 (B v ⌐ A) => (⌐ C Λ D)

P2 (D v E) => ⌐ F / ∴ (A Λ ⌐ F)

1. Let us assume that (A Λ ⌐ F) = 0
2. Out of three instances in which any conjunction is false, let us consider first instance.
3. The conclusion is false when A = 0 and ⌐ F = 0
4. P2 is true iff D v E is false
5. D v E = 0 iff D = 0 and E = 0
6. If D = 0 then (⌐ C Λ D) = 0 irrespective of the truth-value which ⌐ C takes
7. P1 is true iff (B v ⌐ A) = 0 (from 6)
8. ∴ ⌐ A = 0 (from 7)
9. 3 and 8 violate the law of contradiction because both A and ⌐ A cannot be false simultaneously.
10. ∴ A Λ ⌐ F

However, if we consider second instance in which we assume that A = 1 and ⌐ F = 0, then we get different result.

1. A = 1 and ⌐ F = 0
2. P2 = 1 iff D v E = 0
3. D v E = 0 iff D = 0 and E = 0
4. If D = 0 then (⌐ C Λ D) = 0 irrespective of the truth-value which ⌐ C takes
5. P1 = 1 iff (B v ⌐ A) = 0 (from 4)
6. ⌐ A must be 0
7. A = 1 if ⌐ A = 0
8. 7 and 1 are compatible ∵ when A =1, ⌐ A must be 0.
9. ∴ A and ⌐ F = 0

Since in one instance our assumption is wrong and in second instance it is correct, this argument is neither tautological nor contradictory. An argument is said to be contingent when in at least one instance it is true and in atleast one instance it is false. Therefore this argument is called contingent and to arrive at this conclusion we need not consider the result of third circumstance. Therefore it is invalid and to confirm the status of this type of argument at least two instances are necessary.

(The student is advised to consider the third instance in which the conclusion is assumed to be false and then work out the problem.)

It is evident that the method of reductio ad absurdum, when applied to conjunctive conclusion, makes the construction of proof lengthy which renders it the last choice. Secondly, this method succeeds in showing that the truth-table method is primitive because it can be easily shown that ultimately, any other method directly receives support from the truth-table method. It may be noted the rules of inference and replacement derive their authority from truth-table method only. Consider the rule of C.D. which is of the form

{(p => q) Λ (r => s) Λ (p v r)} => (q v s). We shall construct the truth-table to show that this is a tautology.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Sl. No.** | p | q | r | s | **{(p => q)** | Λ | **(r => s)** | Λ | (p v r)} | **=>** | **(q v s)** |
| **1** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **2** | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| **3** | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| **4** | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| **5** | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **6** | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **7** | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 |
| **8** | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 |
| **9** | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 |
| **10** | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| **11** | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| **12** | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **13** | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 |
| **14** | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| **15** | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 16 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 |

In the truth-table method the implication which precedes the consequence component is called the main column. In this table the 11th column is the main column. We notice that in this column, in all 16 instances the truth-value is 1. Therefore the rule is a tautology.

Reductio ad absurdum method makes another critical point more than obvious. If any argument is tautological, then it is logically impossible to assign the truth-values (without landing in self-contradiction) in such a way that the conjunction of premises takes the value 1 while the conclusion takes the value 0. It shows that the truth-values cannot be assigned in a random manner to the components of the statements which constitutes the argument.

**Check Your Progress**

**Note:** a) Use the space provided for your answer.
b) Check your answers with those provided at the end of the unit.

1. What is the advantage of Indirect Proof? Substantiate your answers.

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2. Briefly explain the difference between the rule of conditioned proof and the strengthened rule.

-------------------------------------------------------------------------------------------

3. What is the specialty of combining truth-table method with reductio ad absurdum? Construct an argument using symbols and by applying the methods of truth-table and reductio ad absurdum show that it is a tautology.

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## 3.7 LET US SUM UP

When the conclusion is conditional, formal method does not help. There are three types of conditional statements. There are two kinds of rules of C.P. Indirect Proof is not new to mathematics. Here we reason out in reverse direction. Strengthened rule makes the conclusion independent of assumption.

## 3.8 KEY WORDS

**Reductio ad absurdum:** *Reductio ad absurdum* (Latin: reduction to the absurd) is a form of argument in which a proposition is disproved by assuming the opposite of what is to be proved and deducing its implications to absurd , i.e., self-contradictory consequence.

**Tautology:** A tautology is a series of statements connected logically which is true in all instances. *Contradiction*: it is a form of statement which is false in all instances or whose truth table will have only false substitution instances. *Contingent statements* will have both true and false substitution instances in its truth table.

## 3.9 FURTHER READINGS AND REFERENCES

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## 3.10 ANSWERS TO CHECK YOUR PROGRESS

1. When there are more statements, the formal method of proof becomes unwieldy. In such circumstances I.P. provides a shorter route, sometimes providing proof in one line.

2. When the rule of C.P. is applied, always the antecedent of the conclusion is assumed. However when strengthened rule of C.P. is applied this restriction vanishes. Secondly when the antecedent of the conclusion is assumed invariably it has to be justified by writing C.P. on the R.H.S. adjacent to it. On the other hand, in the case of the strengthened rule a bent arrow is used, the extended part of which marks the limits of assumption. The arrow is a substitute for writing C.P.

3. to self-contradiction. When the truth-table method is applied, we proceed from premises to the conclusion. However, when it is combined with I.P. in order to show that the argument is valid, we proceed from the conclusion and assign '0' value and we proceed to show that it leads

## UNIT 4 QUANTIFICATION

**Contents**



## 4.0 OBJECTIVES

In this unit, we propose to introduce you:

- to a new set of rules to test the validity of arguments, which consist of general and singular propositions.
- to all the rules involved in testing the validity of arguments.
- to understand Aristotle's theory of syllogism against the background of symbolic logic.
- to the application of the new class of rules.

## 4.1 INTRODUCTION

Broadly speaking, there are two types of arguments: arguments consisting of statements, which are truth-functionally compound and arguments, which are neither truth-functional nor compound. This Chapter deals with the latter kind of arguments. Logic, which deals with this branch, is called predicate logic or quantification logic. It is a system of deductive logic that combines the analysis of terms with the analysis of statements by making use of the logical properties of quantifiers. Generally, this type of argument consists of two kinds of statements, called *general and singular*. All propositions accepted by traditional logic belong to these two categories.

Both universal and particular propositions are called general because in these two kinds subject is general term, like men, horses, plants, etc. However, in a singular proposition, the subject refers to a definite individual. The individual may be a human being like 'Tendulkar' or an object like 'the farthest planet from the sun'. The difference between the truth-functional statements on the one hand, and general or singular propositions on the other, is that none of the techniques discussed so far, helps us when arguments consisting of general or singular statements are analyzed. Since quantifying expressions are involved in such statements, quantification is another technique used in our mission to subject these propositions to close scrutiny. Traditional logic or analysis of categorical proposition is the take-off point for quantification logic. Quantity of proposition and subject-predicate relation form the base.

While subject of proposition stands for any individual, predicate stands for the attributes an individual may or may not possess. These individuals and attributes are denoted by lower case letters and upper case letters respectively. With regard to lowercase letters there is one restriction. Only letters from ‘a’ to ‘w’ are used to denote individuals. These are individual constants. Generally, the practice is to choose the first letter of the term to designate the individual. Therefore term like Tendulkar, Dhoni, etc, are represented as t, d, etc. While their attributes like cricketer, swimmer, politician, etc. are designated by C, S, P, etc., by using upper case letters. However, when ‘politician’ becomes subject of a proposition it is designated by ‘p’. In logic, common noun may be subject or predicate. ‘Tendulkar is a cricketer’ is an example for common noun being used as attribute. Symbolically, it becomes ‘Ct’: it is a symbolized statement. First we write the symbol for attribute. This is followed by the symbol for subject. Such a statement can be true or false.

When ‘x’ is used for individual constant, then we have *propositional function*, which is neither true nor false. For example, ‘Bx’ would be a statement like ‘x is brave’. The process of obtaining propositions from propositional function is called ‘*instantiation*’. Thus we can say, ‘Chandran is brave’ – it is a proposition we obtain from the propositional function ‘Bx’. Accordingly, propositional function is an expression that contains one or more individual variables, such that when all its individual variables are replaced by individual constants the result is a symbolized statement. The symbol ‘y’ has a special role to play. It is used to denote an arbitrarily selected individual. In quantification, negation has the same symbol.

## 4.2 QUANTIFICATION: IT’S MEANING

An important aspect of quantification is the substitution of instances. There are two ways in which substitution is being made. In the case of singular proposition, substitution of any individual constant ranging from ‘a’ to ‘w’ can be made to ‘x’ which is known as individual variable; this process is, as we have just seen, instantiation. Another method is through *generalization*. Accordingly, the process of quantification takes place when the given proposition is general. A general proposition is of two types; universal and particular. So we have two quantifiers denoting these two types. Quantifiers are symbols that are used to represent quantifying expression such as everyone / everything / all or someone / something. Thus there are universal or existential quantifiers. In symbols they are respectively as follows: ‘(x)’ and ‘(∃x)’. Since they may be affirmative or negative, we have four kinds of propositions, which are represented as follows:

1. All Indians are mortal. (x) Mx
2. No Indians are mortal. (x) ⌐ Mx
3. Some Indians are moral. (∃x) Mx
4. Some Indians are not mortal. (∃x) ⌐Mx

The symbols used on the right hand side need some explanation.

The symbol (x) is expanded in several ways. It can read ‘for all values of x’ or ‘Given any x’ or simply ‘for every x’, etc., where ‘x’ stands for individual constant ‘Indians’ and ‘M’ stands for ‘mortal’. Therefore ⌐ M x is read ‘x is not mortal’. The symbol (∃x) is read ‘there exists at least one x such that ….' ( ) is called universal quantifier and ∃ is called existential quantifier. If we substitute I (Indians) or P (Pakistanis) for x then we get a proposition, which may be true or false.

Just as x is used as individual variable to denote the subject, two Greek letters 'Φ' (Phi) and 'Ψ' (Psi) are used to denote predicate. So they are called predicate variables. Using these variables, A, E, I and O propositions are represented as follows:

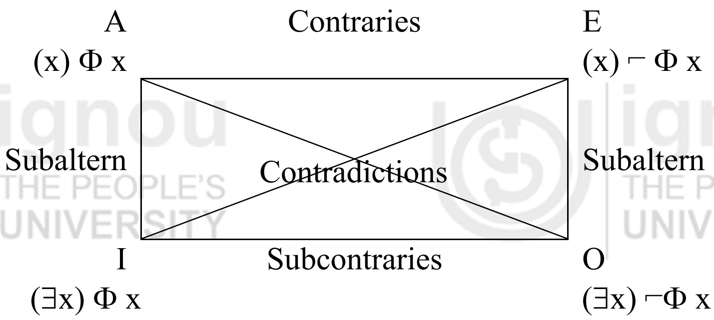
| | | | |
|---|---|---|---|
| 1. | All Indians are mortal | (A) | $(x)\ \Phi x$ |
| 2. | No Indians are mortal | (E) | $(x) \neg \Phi x$ |
| 3. | Some Indians are mortal | (I) | $(\exists x)\ \Phi x$ |
| 4. | Some Indians are not mortal | (O) | $(\exists x) \neg \Phi x$ |

Using class membership relation, general propositions are represented as follows:

1. $(x)\ \Phi x \equiv (x)\{x \in \Phi \Rightarrow x \in \Psi\}$ Where ϵ is read 'element of'
2. $(x) \neg \Phi x \equiv (x)\{x \in \Phi \Rightarrow x \notin \Psi\}$ Where ∉ is read 'not an element of'
3. $(\exists x)\ \Phi x \equiv (\exists x)\{x \in \Phi \wedge x \in \Psi\}$
4. $(\exists x) \neg \Phi x \equiv (\exists x)\{x \in \Phi \wedge x \notin \Psi\}$

## 4.3 LOGICAL RELATIONS INVOLVING QUANTIFIERS

Our study begins with traditional square, which does not need any explanation. We know how A, E, I and O are denoted by quantification. Let us replace A, E, I and O by these quantifiers in the square:




With this background, we represent logical relations, viz., equivalence and contradiction as follows:

1. Equivalence:
   1) $(x)\ \Phi x \equiv \{\neg (\exists x) \neg \Phi x\}$
   2) $(x) \neg \Phi x \equiv \{\neg (\exists x)\ \Phi x\}$
   3) $(\exists x)\ \Phi x \equiv \{\neg (x) \neg \Phi x\}$
   4) $(\exists x) \neg \Phi x \equiv \{\neg (x)\ \Phi x\}$

2. Contradiction:

| | | |
|---|---|---|
| 1) | $(x)\ \Phi x$ | $(\exists x) \neg \Phi x$ |
| 2) | $(x) \neg \Phi x$ | $(\exists x)\ \Phi x$ |
| 3) | $(\exists x)\ \Phi x$ | $(x) \neg \Phi x$ |
| 4) | $(\exists x) \neg \Phi x$ | $(x)\ \Phi x$ |

When we use predicate variable, the propositional forms are expressed as follows:

| | | | |
|---|---|---|---|
| 1) | $(x)\ \Phi x$ | $\equiv$ | $(x)\{\Phi x \Rightarrow \Psi x\}$ |
| 2) | $(x) \neg \Phi x$ | $\equiv$ | $(x)\{\Phi x \Rightarrow \neg \Psi x\}$ |
| 3) | $(\exists x)\ \Phi x$ | $\equiv$ | $(\exists x)\{\Phi x \wedge \Psi x\}$ |
| 4) | $(\exists x) \neg \Phi x$ | $\equiv$ | $(\exists x)\{\Phi x \wedge \neg \Psi x\}$ |

When we represent A, E, I & O with this new set, their equivalent forms also undergo changes.

| | | | |
|---|---|---|---|
| 1) | $(x)\{\Phi x \Rightarrow \Psi x\}$ | $\equiv$ | $\neg(\exists x)\{\Phi x \wedge \neg \Psi x\}$ |
| 2) | $(x)\{\Phi x \Rightarrow \neg \Psi x\}$ | $\equiv$ | $\neg(\exists x)\{\Phi x \wedge \Psi x\}$ |
| 3) | $(\exists x)\{\Phi x \wedge \Psi x\}$ | $\equiv$ | $\neg (x)\{\Phi x \Rightarrow \neg \Psi x\}$ |
| 4) | $(\exists x)\{\Phi x \wedge \neg \Psi x\}$ | $\equiv$ | $\neg (x)\{\Phi x \Rightarrow \Psi x\}$ |

If negations inserted behind the quantifiers on the RHS are removed, then automatically they become contradictions of respective propositions.

A predicate like mortal is called simple predicate because the propositional function which, is used, has some true substitution instances and some false substitution instances. All substitutions to variable are called ‘substitution instances’. When simple predicates are negated, such formulas or statement forms ‘normal-form formula’.

## 4.4 QUANTIFICATION RULES

The rules of inference and replacement are augmented further with the addition of four more rules; universal instantiation (UI), universal generalization (UG), existential instantiation (EI) and existential generalization (EG). With the help of these rules and rules of inference and replacement any argument consisting of general or singular propositions or both can be tested. Before we apply these rules to test the validity of arguments, it is necessary that we know what these rules mean.

1. Universal Instantiation (UI): This rule says that any substitution instance of a propositional function can be validly deduced from a universal proposition. A propositional function always consists of variable ‘x’. Therefore any instance which is a substitution for ‘x’ must be a constant from ‘a’ through ‘w’. These letters signify subject in traditional sense, and in modern sense, an ‘instance of a form’. To transform such proposition ‘x’ is replaced by another Greek letter ‘ν’ (*nu*) when the function is universal quantifier, then ‘ν’ becomes universal instantiation.

$\underline{(x)\ \Phi x}$
$\therefore \Phi \nu$ (where ‘ν’ is any individual symbol)

2. Universal Generalization (UG): This rule helps us to proceed to generalization after an arbitrary selection is made to substitute for 'x'. In UG 'arbitrary selection' is very important because as the name itself suggests, generalization always proceeds from individual instance. And there is choice involved. In this sense, selection is 'random' or arbitrary. The letter 'y' is the symbol of 'arbitrary' selection. This process is called generalization because the conclusion is a universal proposition. If we recall the traditional rules of syllogism, universal conclusion follows from universal premise only. Therefore the process is from universal to universal through an individual. When 'y' replaces 'x' there is generalization. When universal quantifier describes the proposition, it becomes.

$\underline{\Phi y}$
$\therefore (x)(\Phi x)$ (where 'y' refers to any arbitrarily selected individual)

3. Existential Instantiation (EI): This rule is applicable when the proposition has existential quantifier and any symbol ranging from a through w is used as a substitute for the individual variable x. We infer the truth of any substitution instance from existential quantification. However, this rule has a clause. The constant, say 'a' which we use to substitute for x should not have occurred any where earlier in that context. It only means that in the same argument EI cannot be used twice when the substitution instance is only one.

$\underline{\Phi \nu}$
$\therefore (\exists x)\ \Phi x$ (where 'ν' is any individual symbol)

4. Existential Generalization (EG): This rule states that from any true substitution instance of a propositional function, an existential quantification of that function can be validly deduced. In other words, an individual constant which appears in earlier steps, is replaced by x in the conclusion.

$\underline{(\exists x)\ \Phi x}$
$\therefore \Phi \nu$ (where 'ν' is any individual constant other than y that has no prior occurrence in the context)

We should know why there is restriction on the use of EI. Suppose that 'a' is the constant whose existence is definite. We are not sure whether there is any other constant. In the earlier step 'a' is regarded as 'b'. The fact that 'a' is 'b' is not adequate enough to conclude that a is c in some other step when there is no reference of any to it in the premise. Since the logical constant a is used in existential mode, it is mandatory that EI should be used in the very first step of the proof. If it occupies any other position, then it is wrong.

## 4.5 TESTING THE VALIDITY OF SYLLOGISM

It is a matter of great interest to know that the rules of quantification project syllogism in a new perspective, which helps us to abandon the rule of distribution of terms, which is not only cumbersome in presentation but also time consuming. Further, quantification rules can be used to test non-syllogistic arguments also subject to the condition that only general and singular propositions find place in such arguments. Let us use the following arguments to illustrate these rules.

1. 1). All Indians are Asians.
   2). Tendulkar is an Indian.
   3). ∴Tendulkar is an Asian.

   This is symbolized as follows: (x) (Ix=>Ax)
   It
   ∴ At

The formal proof is constructed as follows:

| | | | | |
|---|---|---|---|---|
| 1). | (x)(Ix => Az) | | | |
| 2). | It | /∴At | | |
| 3). | It => At | | 1, | U.I. |
| 4). | ∴At | | 3, 2, | M.P. |

In this particular argument only one premise is general. However, the argument may consist of only general propositions in which case slightly different procedure has to be followed. Consider this argument.

2. 1) All politicians are voters.
   2) All ministers are politicians.
   3) ∴All ministers are voters.

   When symbolized it becomes:

   1) (x)(Px => Vx)
   2) (x)(Mx => Px) /∴ (x)(Mx => Vx)

   The formal proof is as follows:

| | | | |
|---|---|---|---|
| 1) | (x)(Px=>Vx) | | |
| 2) | (x)(Mx=>Px) /∴ (x){Mx=>Vx} | | |
| 3) | Pa => Va | 1, | U.I. |
| 4) | Ma => Pa | 2, | U.I. |
| 5) | Ma => Va | 4, 3, | H.S. |
| 6) | ∴ (x)(Mx => Vx) | 5, | U.G. |

When the individual variable x is instantiated by any constant, then quantifier goes. We do not quantify individual or individuals. Now coming to the $6^{th}$ step, it may be mentioned that if one substitution instance is true for a given structure then all substitution instances must be true for that structure. Further the universal quantification of a propositional function is true if and only if all substitution instances are true. (The $6^{th}$ line is not a part of the proof)

In the third and the fourth steps we have applied universal instantiation because two premises are universal and we have substituted the constants for variables.

UG can be applied in the following manner. Add the sixth line to the proof system after we replace x by y at all stages. Then we have the application of UG

| | | | |
|---|---|---|---|
| 1) | (x){Px => Vx} | | |
| 2) | (x){Mx => Px} | /∴ (x){Mx => Vx} | |
| 3) | Py => Vy | 1, | U.I. |
| 4) | My => Py | 2, | U.I. |
| 5) | My => Vy | 3, 4, | H.S. |
| 6) | ∴ (x){Mx => Vx} | 5 | U.G. |

These two examples suggest that while testing the validity of arguments UI has to be used necessarily though EI may not be necessary. The situation is similar to the traditional formation of rules of syllogism, which hint that without particular propositions it is possible to construct a valid argument, but not without universal propositions.

Now consider an argument, which has a particular proposition. Since one proposition is particular, it is imperative that the conclusion must be particular.

3. 1) All politicians are Voters.
   2) Some ministers are politicians.

   ∴Some ministers are Voters.

By now the method of symbolization should be familiar.

| | | | |
|---|---|---|---|
| 1) | (x){Px => Vx} | | |
| 2) | (∃x{Mx Λ Px} | /∴ (∃x{Mx Λ Vx} | |
| 3) | Ma Λ Pa | 2, | E.I. |
| 4) | Pa => Va | 1, | U.I. |
| 5) | Pa Λ Ma | 3, | Com. |
| 6) | Pa | 5, | Simp. |
| 7) | Ma | 5, | Simp. |
| 8) | Va | 4, 6, | M.P. |
| 9) | Ma Λ Va | 7, 8, | Conj. |
| 10) | ∴ (∃x) (Mx Λ Vx) | 9, | I.G. |

Let us examine why the restriction of EI must be honoured. Consider a fallacious argument.

1) Some animals are herbivorous.
2) Some animals are men.

∴Some men are herbivorous.

When symbolized the argument becomes:

| | | | | |
|---|---|---|---|---|
| 1) | (∃x){Ax Λ Hx} | | | |
| 2) | (∃x){Ax Λ Mx} | /∴(∃x)(Mx Λ Hx) | | |
| 3) | Aa Λ Ha | 1, | E.I. | |
| 4) | Aa Λ Ma | 2, | E.I. | (Error) |

$4^{th}$ Step is erroneous. The second premise tells us that there is at least one thing that is both an animal and herbivorous. It does not permit us to conclude that it should also be regarded as man. Therefore a second use of EI leads to error.

## 4.6 MULTIPLY GENERAL PROPOSITIONS

There are two types of general proposition; singly general and multiply general. If a general proposition has only one quantifier, then it is called *singly general*. Until now, we considered only propositions of former kind. If a general proposition consists of two or more than two quantifiers, then such a proposition is called *multiply general propositions*. Consider, for example, this proposition:

"If all Indians play cricket, then there are at least some Asians who play cricket."

Its symbolization is as follows:
1) All Indians play cricket: (x){Ix => Px}
2)There are at least some Asians who play cricket:(∃x){Ax Λ Px}

Now the symbolization of the whole sentence is as follows:

{(x)(x => Px)} => {(∃x)(Ax Λ Px)}

Depending upon the complexity of the given statement quantifiers may occur any number of times.

## 4.7 THE STRENGTHENED RULE OF C.P. AND QUANTIFICATION

In the previous unit, we learnt that assumption is different from conditional proof and that assumption does not include the conclusion, which depends solely on the premise. A few examples will illustrate how an argument can be tested using these techniques.

1. 1) (x)[Cx => Dx]
   2) (x)[Ex => ⌐ Dx]
   ∴ (x)[Ex => ⌐ Cx]

The argument is written in standard form;

| | | | |
|---|---|---|---|
| 1) | (x)[Cx => Dx] | | |
| 2) | (x)[Ex => ⌐ Dx] | / ∴ (x)[Ex => ⌐ Cx] | |
| → 3) | Ey | | |
| 4) | Cy => Dy | 1, | U.I. |
| 5) | Ey => ⌐ Dy | 2, | U.I. |
| 6) | ⌐Dy | 5, 3, | M.P. |
| 7) | ⌐Cy | 4, 6, | M.T. |
| 8) | Ey => ⌐ Cy | 3, 7, | C.P. |
| 9) | (x)[Ex => ⌐ Cx] | 9, | U.G. |

From (1) two aspects become clear. The limit of assumption ends, when CP is used. So next step does not depend upon this assumption. Second, since we are making an assumption, in place of 'x' only 'y'; an arbitrary chosen symbol can be used. This explanation holds good whenever the strengthened rule of CP is used.

2.

| | | | |
|---|---|---|---|
| 1) | (x)[Nx => Oy] | | |
| 2) | (x)[Px => ⌐ Ox] | /∴ (x) {(Nx Λ ⌐Px) => Ox} | |
| → 3) | Ny | | |
| 4) | Ny => Oy | 1, | U.I. |
| 5) | Py => ⌐ Oy | 2, | U.I. |
| 6) | Oy | 4, 3, | M.P. |
| 7) | ⌐ Py | 5, 6, | M.T. |
| 8) | Ny Λ ⌐ Py | 3, 7, | Conj. |
| 9) | (Ny Λ ⌐ Px) => Oy | 8, 6, | C.P. |
| 10) | (x){ (Nx Λ ⌐ Px) => Ox} | 9, U.G. | |

## 4.8 PROVING INVALIDITY

The cardinal principle underlying the classification of arguments into good and bad is that true premises do not yield false conclusion. The easiest way of identifying the false conclusion in association with true premises is the method of assigning the truth-values to the components of statements. When the method of truth-values is extended to arguments with quantifiers one requirement has to be satisfied. We have to consider a nonempty model which is similar to a nonempty set. This model is the locus of our discussion. An argument involving quantifiers is valid if and only if to every nonempty model corresponds a logically equivalent and valid truth-functional argument. Similarly, an argument is invalid if there is a nonempty model to which corresponds a logically equivalent and invalid truth-functional argument. The crux of the matter is only this; an argument consisting of quantifiers is valid if and only if its truth-functional mode is valid and invalid if and only if its truth-functional mode is invalid. Since there is recourse to truth-functional mode, it is necessary to know how statements with quantifiers can be reduced to truth-functional compound statements. The very same truth-conditions which determine the truth-value of compound propositions also determine the truth-conditions of corresponding propositions with quantifiers.

In the beginning of this section, we mentioned that an argument with quantifiers is valid if there is 'at least' one individual. It only means that there can be any number of individuals in a nonempty model. Suppose that there are only three men in the model of men, viz. a, b and c. In such a case the proposition 'A' can be represented in the following manner.

1. (x) (Φx) ≡ (Φa Λ Φb Λ Φc)

The LHS is true if and only if Φa is true, Φb is true and Φc is true. If any one of them is false, then the LHS is false. Similarly, the proposition 'E' becomes

2. (x) (⌐ Φx) ≡ (⌐ Φa Λ⌐ Φb Λ⌐ Φc)

   If a, b and c are the only men in the model of men, then as in the previous case, in the present case also the LHS is true if and only if everyone of the three components is true. If any one of them is false then LHS also is false.

While the propositions with universal quantifiers are translated to the conjunction mode, those with existential quantifiers are reduced to the disjunction mode. If we persist with the same model, then

3. (∃x) (Φx) ≡ (Φa v Φb v Φc)
4. (∃x) (¬ Φx) ≡ (¬ Φa v ¬ Φb v ¬ Φc)

From these four equations, it is clear that the truth status of propositions with quantifiers is determined by the truth-conditions of compound proposition. For example, consider (1). Even if one component on the RHS is false, then the LHS also turns out to be false. This is because conjunction is false when any component is false and in disjunction when any one component is true, the LHS is true. This type of relation is in perfect consonance with the definition of universal and existential quantifiers.

Suppose that there is only one individual. Then two corollaries follow from this supposition, which are as follows.

1. (x) (Φx) ≡ Φa ≡ (∃x) (Φx)
2. Since there is only one true substitution instance (SI) to x, viz. a, we do not derive Φa from (x) (Φx)

When there is only one individual any logical difference between universal and existential quantifiers also ceases to operate.

Logically, there is a qualitative difference between a model containing only one individual and another model containing two or more than two individuals. (For the sake of convenience let us call the first model monadic and the second one polyadic model. If there are two individuals then the model is dyadic and if there are more than two then triadic and so on). There is a qualitative difference because in a monadic model an invalid argument may correspond to a valid truth-functional argument whereas the very same argument in any other model may correspond to an invalid truth-functional argument. Let us consider an argument which is invalid from traditional angle.

1.

All politicians are lawyers.
All judges are lawyers.
∴ All judges are politicians.

1. (x) [Px => Lx]
2. (x) [Jx => Lx] / ∴ (x) Jx => Px

Since there is only one SI, this argument is logically equivalent to

3. p1:[Pa => La]
4. p2:[Ja => La] / ∴ Ja => Pa

In a monadic model (x) (Φx) ≡ Φa ≡ (∃x) (Φx)
∴ The argument is logically equivalent to

5. Pa Λ La
6. Ja Λ La / ∴ Ja Λ Pa

If we assign the value 0 to any one of the components of the conclusion then not only the conclusion is false but also one of the premises becomes false. However, according to definition, the premises must be true. It is logically impossible to derive a false conclusion from true premises. Therefore in this case the argument is valid. However, the same argument is invalid in a dyadic model. Before we consider an example for an argument in a dyadic model, let us consider the structure of the model.

[(x) (Φx)] ≡ [Φa Λ Φb]
[(x) ⌐ (Φx)] ≡ [⌐ Φa Λ⌐ Φb]
(∃x) (Φx) ≡[Φa v Φb]
(∃x) ⌐ (Φx) ≡[⌐ Φa v ⌐ Φb]

Where a and b are two individuals who (or which) are the members of a dyadic model

2. Now let us symbolise the previous argument
   1. p1: (x) [Px => Lx]
   2. p2: (x) [Jx => Lx] / ∴ (x) Jx => Px

   Since we are considering a dyadic model the symbolic presentation is logically equivalent to

   3. (Pa => La) Λ (Pb => Lb)
   4. (Ja => La) Λ (Jb => Lb) / ∴ (Ja => Pa) Λ (Jb => Pb)

   Assign 0 to Pa and 1 to the rest. The result can be computed as follows

   5. (Pa => La) Λ (Pb => Lb)
      0 1 1 1 1 1 1
   6. (Ja => La) Λ (Jb => Lb) / ∴ (Ja => Pa) Λ (Jb => Pb)
      1 1 1 1 1 1 1 1 0 0 0 1 1 1

   The conjunction of the truth-values which are boxed in 5 and 6 yields true premises whereas the conclusion is false. Hence the argument is invalid. This result can be generalised to include other polyadic models with 3 or more than 3 members. Whatever holds good to a dyadic model in this case also holds good to any other polyadic model. To become familiar with this method let us work with some more problems.

3. (x) (Dx => ⌐ Ex)
   (x) (Ex => Fx) / ∴(x) (Fx => ⌐ Dx)

   Let us restrict this argument to a dyadic model. If this argument is invalid in this model, then it is invalid in all other polyadic models. The logically equivalent form of 3 is as follows.

   1. (Da => ⌐ Ea) Λ (Db => ⌐ Eb)
   2. (Ea => Fa) Λ (Eb => Fb) / ∴(Fa => ⌐ Da) Λ (Fb => ⌐ Db)

   Assign 0 to ⌐ Da. In accordance with the law of contradiction Da = 1. Similarly, ⌐ Db is assigned 0. Therefore Db = 1. Assign 1 to ⌐ Ea. Ea takes 0. Assign 1 to ⌐ Eb. Eb takes 0. Assign 1 to Fa and Fb. The result can be computed as follows.

   3. (Da => ⌐ Ea) Λ (Db => ⌐ Eb)
      1 1 1 1 1 1 1
   4. (Ea => Fa) Λ (Eb => Fb) / ∴(Fa => ⌐ Da) Λ (Fb => ⌐ Db)
      0 1 1 1 0 1 1 1 0 0 0 1 0 0

In this argument also the conjunction of the truth-values boxed in 3and 4 yields true premises whereas the conclusion is false. Hence the argument is invalid. This result can be generalised to include other polyadic models with 3 or more than 3 members. Whatever holds good to a dyadic model in this case also holds good to any other polyadic model.

4.

1. (∃x) (Jx Λ Kx)

2. (∃x) (Kx Λ Lx) / ∴(∃x) (Lx Λ Jx)

We shall consider this argument also in a dyadic model. This is logically equivalent to

3.(Ja Λ Ka) v (Jb Λ Kb)

4. (Ka Λ La) v (Kb Λ Lb) /∴(La Λ Ja) v (Lb Λ Jb)

There is a difference between this argument and the previous arguments. In this argument the premises and conclusion are disjunctive unlike the previous arguments which have conjunctive statements. The difference is due to quantifiers. In case of universal quantifiers conjunction is the connective whereas in case of existential quantifiers disjunction is the connective.

Assign the truth-values as follows; 0 to La and Jb and 1 to the rest. The result is computed as follows.

5.(Ja Λ Ka) v (Jb Λ Kb)
  1 1 1   1  0 0 1
6. (Ka Λ La) v (Kb Λ Lb) /∴(La Λ Ja) v ( Lb Λ Jb)
  1 0 0   1  1 1 1   0 0 1  0  1 0 0

In this argument also the conjunction of the truth-values which are boxed in 5 and 6 yields true premises whereas the conclusion is false. Hence the argument is invalid. This result can be generalised to include other polyadic models with 3 or more than 3 members. Whatever holds good to a dyadic model in this case also holds good to any other polyadic model.

## 4.9 NONSYLLOGISM

All arguments need not be syllogistic even though they consist of two premises and a conclusion. Relational argument is one such example.

1. Bangalore is to the west of Chennai.
   Mangalore is to the west of Bangalore.
   ∴Mangalore is to the west of Chennai.

Aristotelian system does not regard this class of arguments as syllogistic though this can be shown to be valid in symbolic representation, but it results in the distortion of the meaning of statements. If we try to retain the meaning, then it becomes impossible to demonstrate the validity or invalidity, as the case may be.

Apart from relational arguments, there is another class of arguments which consists of more than three terms and propositions. Consider this argument.

Men (1) are both stupid (2) and dishonest (3).
Some men are irritable (4).
∴ Some dishonest persons (3) are irritable (4).

Terms are numbered so there is no confusion. However, the statements are misleading. If we regard a conjunctive proposition as one proposition, then in this argument there are three propositions. Even if the previous statement is accepted the argument cannot be syllogistic because there are four terms. If we give priority to simple propositions then the first premise has two simple propositions. Then we will have four propositions. Therefore this type of argument is classified as nonsyllogistic. To test this kind of argument we do not require any additional rule. Proper symbolization of this class of argument is important. The symbolization is as follows:

1. (x) [Mx => (Sx Λ Dx)]
2. (∃x) [Mx Λ Ix] / ∴ (∃x) (Ix Λ Sx). Its formal proof:

| | | |
|---|---|---|
| 3. | [Ma Λ Ia] | 2, E. I. |
| 4. | Ma => (Sa Λ Da) | 1, U. I. |
| 5. | Ma | 3, Simp. |
| 6. | (Sa Λ Da) | 4, 5, M. P. |
| 7. | Sa | 6, Simp. |
| 8. | Ia | 3, Simp. |
| 9. | Ia Λ Sa | 8, 7, Conj. |
| 10. | (∃x) (Ix Λ Sx) | 9, E.G. |

The status of (1) calls for our attention. Had the first premise been regarded as a conjunctive proposition, then (1) ought to have been symbolized as
11. Sm Λ Dm
It is a well known fact that conjunction does not have any equivalent form. Therefore (1) is not equivalent to (11).

Consider another statement, which has a very different structure.

Americans and Germans are pioneers in science.

This statement actually means that a pioneer in science may be an American or a German. Surely, it does not mean at a pioneer in science is both an American and a German. Hence when this innocuous statement is translated into logical language, it becomes a disjunctive proposition with exclusive 'Or'. Nor is it a conjunctive proposition of the form
Americans are pioneers in science and Germans are pioneers in science.
This is so because a conjunctive proposition of this form means the same as saying that a pioneer in science is both an American and a German, which is absurd. Consider this argument:
Americans and Germans are scientists.
Some white men are Americans.
Therefore, some white men are scientists.
This argument is symbolized as follows:

| | | |
|---|---|---|
| 1. | (x) [(Ax v Gx) => Sx] | |
| 2. | (∃x) [Wx Λ Ax] | / ∴ (∃x)[Wx Λ Sx] |
| 3. | Wa Λ Aa | 2, E.I. |
| 4. | Aa | 3, Simp. |
| 5. | (Aa v Ga) | 4, Add. |
| 6. | (Aa v Ga) => Sa | 2, U.I. |
| 7. | Sa | 6, 5, M.P. |
| 8. | Wa | 3, Simp. |
| 9. | Wa Λ Sa | 8, 7, Conj. |
| 10. | (∃x)[Wx Λ Sx] | 9, E.G. |

In one particular sense, nonsyllogistic arguments are more significant than traditional syllogism for the simple reason that in any debate, whether based in science or politics, syllogism is seldom used. Application of nonsyllogistic arguments is widespread and more useful. Therefore there is greater need to become familiar with nonsyllogistic arguments.

**Check Your Progress**

**Note:** a) Use the space provided for your answer.
b) Check your answers with those provided at the end of the unit.

I. Construct formal proofs of validity.

1. 1) $(x)[Qx \Rightarrow Rx]$
   2) $(\exists x)[Qx \vee Rx]$
   $\therefore (\exists x) Rx$

-------------------------------------------

2. 1) $(x)[Sx \Rightarrow (Tx \Rightarrow Ux)]$
   2) $(x)[Ux \Rightarrow (Vx \wedge Wx)]$
   $\therefore (x)[Sx \Rightarrow (Tx \Rightarrow Vx \wedge Wx)]$

-------------------------------------------------------------

3. 1) $(x)[Dx \Rightarrow \neg Ex]$
   2) $(x)[Fx \Rightarrow Ex]$
   $\therefore (x)[Fx \Rightarrow \neg Dx]$

----------------------------------------------------------

4. 1) $(\exists x)[Jx \wedge Kx]$
   2) $(x)[Jx \Rightarrow Lx]$
   $\therefore (\exists x)[Lx \wedge Kx]$

------------------------------------------------------------

## 4.10 LET US SUM UP

Quantification is another set of rules, which augments the logical tools of test. It applies to arguments, which consist of general and singular propositions. Quantification rules must be used in conjunction with the rules of inference and replacement.

## 4.11 KEY WORDS

**Dyadic:** Dyadic is that which is composed of two sets of objects say *A* and *B;* if three sets or elements, then it is known as *triadic*; if four, then *tetradic* and if five, then *pentadic*.

**Polyadic:** Polyadic is that which comprises of many elements.

## 4.12 FURTHER READINGS AND REFERENCES

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## 4.13 ANSWERS TO CHECK YOUR PROGRESS

| | | |
|---|---|---|
| 1) | (x) [Qx => Rx] | |
| 2) | (∃x) [Qx v Rx] | /∴ (∃x) (Rx) |
| 3) | Qa v Ra | 2, E.I. |
| 4) | Qa => Ra | 1, U.I. |
| 5) | Ra | 4, 3, M.P. |
| 6) | (∃x) Rx | 5, E.G. |

2

| | | |
|---|---|---|
| 1) | (x) [Sx => (Tx => Ux)] | |
| 2) | (x) [Ux => (Vx Λ Wx)] | /∴ (x) [Sx => {Tx => (Vx Λ Wx)}] |
| 3) | Sa => (Ta => Ua) | 1, U.I. |
| 4) | Ua => (Va Λ Wa) | 2, U.I. |
| 5) | (Sa Λ Ta) => Ua | 3, Exp. |
| 6) | (Sa Λ Ta) => (Va Λ Wa) | 5, 4, H.S. |
| 7) | Sa => (Ta => (Va Λ Wa)) | 6, Exp. |
| 8) | ∴ (x) [Sx => {Tx => (Vx Λ Wx)}] | 7, U.G. |

3

| | | |
|---|---|---|
| 1) | (x) [Dx => ⌐ Ex] | |
| 2) | (x) [Fx => Ex] | /∴ (x) [Fx => ⌐ Dx] |
| 3) | Da => ⌐ Ea | 1, U.I. |
| 4) | Fa => Ea | 2, U.I. |
| 5) | Ea => ⌐ Da | 3, Trans. |
| 6) | Fa => ⌐ Da | 4, 5, H.S. |
| 7) | ∴ (x) [Fx => ⌐ Dx] | 6, U.G. |

4

| | | |
|---|---|---|
| 1) | (∃x) [Jx Λ Kx] | |
| 2) | (x) [Jx => Lx] | /∴ (∃x) [Lx Λ Kx] |
| 3) | Ja Λ Ka | 1, E.I. |
| 4) | Ja => La | 2, U.I. |
| 5) | Ja | 3, Simp. |
| 6) | Ka | 3, Simp. |
| 7) | La | 4, 5, M.P. |
| 8) | La Λ Ka | 7, 6, Conj. |
| 9) | (∃x) [Lx Λ Kx] | 8, E.G. |